Lévy driven fixed income models

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Lévy term structure models

The Lévy forward rate model (HJM type)

\[ f(t, T) = f(0, T) + \int_0^t \alpha(s, T) \, ds - \int_0^t \sigma(s, T) \, dL_s \]

The Lévy forward process model

\[ F(t, T_j^*, T_{j-1}^*) = F(0, T_j^*, T_{j-1}^*) \exp \left( \int_0^t \lambda(s, T_j^*) \, dL_{T_j^*}^{T_{j-1}^*} \right) \]

The Lévy Libor or market model

\[ L(t, T_j^*) = L(0, T_j^*) \exp \left( \int_0^t \lambda(s, T_j^*) \, dL_{T_j^*}^{T_{j-1}^*} \right) \]

\[ 1 + \delta L(t, T_j^*) = F(t, T_j^*, T_{j-1}^*) \]
Extensions of the Lévy market model

- Multi-currency setting (Eb–Koval (2006))
- Credit risk model (Eb–Kluge–Schönbucher (2006))
- Swap rate model (Eb–Liinev (2006))
- Duality principle (Eb–Kluge–Papapantoleon (2006))
Comparison of estimated interest rates (least squares Svensson)

Termstructure, February 17, 2004
Caplet market data

Euro caplet implied volatility surface on February 19, 2002
Libor rates in a cross currency setting

Discrete tenor structure $T_0 < T_1 < \cdots < T_n < T_{n+1} = T^*$
Accrual periods $\delta = T_{j+1} - T_j$

(m + 1) markets $i = 0, \ldots, m$
0 = domestic market

Want to model the dynamics of the Libor rate $L^i(t, T_{j-1})$ which applies to time period $[T_{j-1}, T_j]$ in market $i$ ($i = 0, \ldots, m$)
We target at the form

$$L^i(t, T_{j-1}) = L^i(0, T_{j-1}) \exp \left( \int_0^t \lambda^i(s, T_{j-1}) dL_s^{i,T_{j}} \right)$$
The driving process

$L^{0,T*} = (L^{0,T*}_1, \ldots, L^{0,T*}_d)$ is a $d$-dimensional time-inhomogeneous Lévy process. The law of $L^{0,T*}_t$ is given by

$$\mathbb{E}[\exp(iu^\top L^{0,T*}_t)] = \exp \int_0^t \theta^{0,T*}_s (iu) \, ds \quad \text{with}$$

$$\theta^{0,T*}_s (z) = z^\top b^{0,T*}_s + \frac{1}{2} z^\top C_s z + \int_{\mathbb{R}^d} \left( e^{z^\top x} - 1 - z^\top x \right) \lambda^{0,T*}_s (dx),$$

where $b^{0,T*}_t \in \mathbb{R}^d$, $C_s$ is a symmetric nonnegative-definite $d \times d$-matrix and $\lambda^{0,T*}_s$ is a Lévy measure.

Integrability:

$$\int_0^{T*} \left( |b^{0,T*}_s| + ||C_s|| + \int_{\{|x| \leq 1\}} |x|^2 \lambda^{0,T*}_s (dx) \right) ds < \infty$$

$$\int_0^{T*} \int_{\{|x| > 1\}} \exp(u^\top x) \lambda^{0,T*}_s (dx) ds < \infty \quad (u \in [-M, M]^d)$$
Description in terms of modern stochastic analysis

$L^{0,T*} = (L_{t}^{0,T*})$ is a special semimartingale with canonical representation

$$L_{t}^{0,T*} = \int_{0}^{t} b_{s}^{0,T*} \, ds + \int_{0}^{t} c_{s} \, dW_{s}^{0,T*} + \int_{0}^{t} \int_{\mathbb{R}^d} x(\mu - \nu_{0,T*})(ds, dx)$$

$(W_{t}^{0,T*})$ is a $\mathbb{P}^{0,T*}$-standard Brownian motion with values in $\mathbb{R}^d$

c_t is a measurable version of the square root of $C_t$

$\mu$ the random measure of jumps of $(L_{t}^{0,T*})$

$\nu_{0,T*}(ds, dx) = \lambda_{s}^{0,T*}(dx) \, ds$ is the $\mathbb{P}^{0,T*}$-compensator of $\mu$

$(L_{t}^{0,T*})$ is also called a process with *independent increments* and *absolutely continuous characteristics* (PIIAC).
Simulation of a Lévy process

$\text{NIG}(10,0,0.100,0)$ on $[0,1]$

$\text{NIG}(10,0,0.025,0)$ on $[1,3]$
The foreign forward exchange rate for date $T^*$ \hspace{1cm} (1)

**Assumption**

*(FXR.1):* For every market $i \in \{0, \ldots, m\}$ there are strictly decreasing and strictly positive zero-coupon bond prices $B^i(0, T_j)(j = 0, \ldots, N + 1)$ and for every foreign economy $i \in \{1, \ldots, m\}$ there are spot exchange rates $X^i(0)$ given.

Consequently the initial foreign forward exchange rate corresponding to $T^*$ is

$$F_{X^i}(0, T^*) = \frac{B^i(0, T^*)X^i(0)}{B^0(0, T^*)}$$
The foreign forward exchange rate for date $T^*$ \hspace{1cm} (2)

Assumption

(FXR.2): For every foreign market $i \in \{1, \ldots, m\}$ there is a continuous deterministic function $\xi^i(\cdot, T^*) : [0, T^*] \to \mathbb{R}^d$.

For every coordinate $1 \leq k \leq d$ we assume

$$(\xi^i(s, T^*))_k \leq \overline{M} \quad (s \in [0, T^*], \ 1 \leq i \leq m)$$

where $\overline{M} < \frac{M}{N + 2}$. 

The foreign forward exchange rate for date $T^*$ (3)

**Assumption**

**(FXR.3):** For every $i \in \{1, \ldots, m\}$ the foreign forward exchange rate for date $T^*$ is given by

$$F_{X_i}(t, T^*) = F_{X_i}(0, T^*) \exp \left( \int_0^t \gamma^i(s, T^*) \, ds + \int_0^t \xi^i(s, T^*)^\top dL_{s,T^*} \right)$$

where

$$\gamma^i(s, T^*) = -\xi^i(s, T^*)^\top b_{s,T^*}^0 - \frac{1}{2} |\xi^i(s, T^*)^\top c_s|^2$$

$$- \int_{\mathbb{R}^d} \left( e^{\xi^i(s,T^*)^\top x} - 1 - \xi^i(s, T^*)^\top x \right) \lambda^{0,T^*}_s(dx)$$

Equivalently

$$F_{X_i}(t, T^*) = F_{X_i}(0, T^*) \mathcal{E}_t \left( \int_0^t \xi^i(s, T^*)^\top c_s \, dW_{s,T^*} + \int_0^t \int_{\mathbb{R}^d} \left( \exp \left( \xi^i(s, T^*)^\top x \right) - 1 \right) (\mu - \nu_{0,T^*})(ds, dx) \right)$$
The foreign forward exchange rate for date $T^*$ (4)

Consequences: $F_{X^i}(\cdot, T^*)$ is a $\mathbb{P}^{0,T^*}$-martingale

$$E_{\mathbb{P}^{0,T^*}} \left[ \frac{F_{X^i}(t, T^*)}{F_{X^i}(0, T^*)} \right] = 1$$

Define

$$\frac{d\mathbb{P}^{i,T^*}}{d\mathbb{P}^{0,T^*}} \bigg|_{\mathcal{F}_t} = \frac{F_{X^i}(t, T^*)}{F_{X^i}(0, T^*)}$$

By Girsanov’s theorem we get a $\mathbb{P}^{i,T^*}$-standard Brownian motion

$$W_{t}^{i,T^*} = W_{t}^{0,T^*} - \int_{0}^{t} c_s \xi^i(s, T^*) \, ds$$

and a $\mathbb{P}^{i,T^*}$-compensator

$$\nu_{i,T^*}(dt, dx) = \exp(\xi^i(t, T^*)^\top x)\nu_{0,T^*}(dt, dx)$$
The Lévy Libor model
Eberlein–Özkan (2005)

Tenor structure \( T_0 < T_1 < \cdots < T_N < T_{N+1} = T^* \)

with \( T_{j+1} - T_j = \delta \), set \( T_j^* = T^* - j\delta \) for \( j = 1, \ldots, N \)

Assumptions

\( (\text{CLM.1}): \) For every market \( i \) and every maturity \( T_j \) there is a bounded deterministic function \( \lambda^i(\cdot, T_j) \), which represents the volatility of the forward Libor rate process \( L^i(\cdot, T_j) \) in market \( i \).

\( (\text{CLM.2}): \) The initial term structure of forward Libor rates in market \( i \) is given by

\[
L^i(0, T_j) = \frac{1}{\delta} \left( \frac{B^i(0, T_j)}{B^i(0, T_j + \delta)} - 1 \right)
\]
Backward Induction (1)

Given a stochastic basis \((\Omega, \mathcal{F}_{T^*}, \mathbb{P}^{0,T^*}, (\mathcal{F}_t)_{0 \leq t \leq T^*})\)

We postulate that under \(\mathbb{P}^{i,T^*}\)

\[
L^i(t, T^*_1) = L^i(0, T^*_1) \exp \left( \int_0^t \lambda^i(s, T^*_1) \, dL^i_{s,T^*} \right)
\]

where

\[
L_{t,T^*}^i = \int_0^t b_{s,T^*}^i \, ds + \int_0^t c_s \, dW^i_{s,T^*} + \int_0^t \int_{\mathbb{R}^d} x(\mu - \nu_{i,T^*})(ds, dx)
\]

with \(W^i_{-,T^*}\) and \(\nu_{i,T^*}\) as given before.
Backward Induction (2)

In order to make \( L^i(t, T_1^*) \) a \( \mathbb{P}^{i,T^*} \)-martingale, choose the drift characteristic \( (b_s^{i,T^*}) \) s.t.

\[
\int_0^t \lambda^i(s, T_1^*) b_s^{i,T^*} \, ds = -\frac{1}{2} \int_0^t |\lambda^i(s, T_1^*) c_s|^2 \, ds
\]

\[
- \int_0^t \int_{\mathbb{R}^d} \left( e^{\lambda^i(s,T_1^*)x} - 1 - \lambda^i(s, T_1^*)x \right) \nu_{i, T^*}(ds, dx)
\]

Transform \( L^i(t, T_1^*) \) in a stochastic exponential

\[
L^i(t, T_1^*) = L^i(0, T_1^*) \mathcal{E}_t(H^i(\cdot, T_1^*))
\]

where

\[
H^i(t, T_1^*) = \int_0^t \lambda^i(s, T_1^*) c_s \, dW_s^{i,T^*} + \int_0^t \int_{\mathbb{R}^d} \left( e^{\lambda^i(s,T_1^*)x} - 1 \right) (\mu - \nu_{i,T^*})(ds, dx)
\]
Backward Induction (3)

Equivalently

$$dL^i(t, T_1^*) = L^i(t-, T_1^*) \left( \lambda^i(t, T_1^*) c_t \, dW_t^i; T^* ight. $$

$$ + \int_{\mathbb{R}^d} \left( e^{\lambda^i(t, T_1^*) x} - 1 \right) (\mu - \nu_{i, T^*})(dt, dx) \right)$$

with initial condition

$$L^i(0, T_1^*) = \frac{1}{\delta} \left( \frac{B^i(0, T_1^*)}{B^i(0, T^*)} - 1 \right)$$
Recall $F_{Bi}(t, T_1^*, T^*) = 1 + \delta L^i(t, T_1^*)$, therefore,
\[
dF_{Bi}(t, T_1^*, T^*) = \delta dL^i(t, T_1^*) = F_{Bi}(t, T_1^*, T^*) = 1 + \delta L^i(t, T_1^*)c_t dW_t^{i, T^*}
\]
\[
= F_{Bi}(t-, T_1^*, T^*) \left( \frac{\delta L^i(t-, T_1^*)}{1 + \delta L^i(t-, T_1^*)} \lambda^i(t, T_1^*) \right) c_t dW_t^{i, T^*}
\]
\[
= \alpha^i(t, T_1^*, T^*)
\]
\[
+ \int_{\mathbb{R}^d} \frac{\delta L^i(t-, T_1^*)}{1 + \delta L^i(t-, T_1^*)} \left( e^{\lambda^i(t, T_1^*)x} - 1 \right) (\mu - \nu_{i, T^*})(dt, dx)
\]
\[
= \beta^i(t, x, T_1^*, T^*) - 1
\]
Define the forward martingale measures associated with $T_1^*$
\[
\frac{d\mathbb{P}^{i, T_1^*}}{d\mathbb{P}^{i, T^*}} = \mathcal{E}_{T_1^*}(M_{i,1}^{i,1})
\]
where
\[
M_{i,1}^{i,1} = \int_0^t \alpha^i(s, T_1^*, T^*) c_s dW_s^{i, T^*} + \int_0^t \int_{\mathbb{R}^d} \left( \beta^i(s, x, T_1^*, T^*) - 1 \right) (\mu - \nu_{i, T^*})(ds, dx)
\]
Backward Induction (5)

Then
\[ W_{t}^{i,T_{1}^{*}} = W_{t}^{i,T_{*}} - \int_{0}^{t} \alpha^{i}(s,T_{1}^{*},T_{*}) c_{s} \, ds \]

is the forward Brownian motion for date \( T_{1}^{*} \) and

\[ \nu_{i,T_{1}^{*}}(dt, dx) = \beta^{i}(t, x, T_{1}^{*}, T_{*}) \nu_{i,T_{*}}(dt, dx) \]

is the \( \mathbb{P}^{i,T_{1}^{*}} \)-compensator for \( \mu \).

Second step

We postulate that under \( \mathbb{P}^{i,T_{1}^{*}} \)

\[ L^{i}(t, T_{2}^{*}) = L^{i}(0, T_{2}^{*}) \exp \left( \int_{0}^{t} \lambda^{i}(s,T_{2}^{*}) \, dL_{s}^{i,T_{1}^{*}} \right) \]

where

\[ L_{t}^{i,T_{1}^{*}} = \int_{0}^{t} b_{s}^{i,T_{1}^{*}} \, ds + \int_{0}^{t} c_{s} \, dW_{s}^{i,T_{1}^{*}} + \int_{0}^{t} \int_{\mathbb{R}^{d}} x(\mu - \nu_{i,T_{1}^{*}}) \, (ds, dx) \]
Backward Induction (6)

Second measure change

\[ \frac{d\mathbb{P}^{i,T_2^*}}{d\mathbb{P}^{i,T_1^*}} = \mathcal{E}_{T_2^*}(M^{i,2}) \]

where

\[ M^{i,2}_t = \int_0^t \alpha^i(s, T_2^*, T_1^*) c_s \, dW^{i,T_1^*}_s \]

\[ + \int_0^t \int_{\mathbb{R}^d} \left( \beta^i(s, x, T_2^*, T_1^*) - 1 \right) (\mu - \nu_{i,T_1^*})(ds, dx) \]

This way we get for each time point \( T_j^* \) in the tenor structure a Libor rate process which is under the forward martingale measure \( \mathbb{P}^{i,T_{j-1}^*} \) of the form

\[ L^i(t, T_j^*) = L^i(0, T_j^*) \exp \left( \int_0^t \lambda^i(s, T_j^*) \, dL^{i,T_{j-1}^*}_s \right) \]
Alternative Backward Induction (1)

Postulate

\[ 1 + \delta L^i(t, T_1^*) = (1 + \delta L^i(0, T_1^*)) \exp \left( \int_0^t \lambda^i(s, T_1^*) \, dL^i_{s, T_1^*} \right) \]

equivalently

\[ F_{Bi}(t, T_1^*, T^*) = F_{Bi}(0, T_1^*, T^*) \exp \left( \int_0^t \lambda^i(s, T_1^*) \, dL^i_{s, T_1^*} \right) \]

In differential form

\[
\begin{align*}
\mathrm{d}F_{Bi}(t, T_1^*, T^*) &= F_{Bi}(t-, T_1^*, T^*) \left( \lambda^i(t, T_1^*) c_t \, dW^i_{t, T_1^*} ight) \\
&\quad + \int_{\mathbb{R}^d} \left( e^{\lambda^i(t, T_1^*) x} - 1 \right) (\mu - \nu_{i, T_1^*})(dt, dx) 
\end{align*}
\]
Define the forward martingale measures associated with $T^*_1$

\[
\frac{d\mathbb{P}^{i,T^*_1}}{d\mathbb{P}^{i,T^*}} = \mathcal{E}_{T^*_1}(\tilde{M}^{i,1})
\]

where

\[
\tilde{M}^{i,1} = \int_0^t \lambda^i(s, T^*_1)c_s \, dW^i_s,T^* + \int_0^t \int_{\mathbb{R}^d} \left( e^{\lambda^i(s, T^*_1)\chi} - 1 \right)(\mu - \nu_{i,T^*})(ds, dx).
\]
Cross-currency Lévy market model

**Domestic Market**
- \( \mathbb{P}^{0}, T^{*} \) -forward measure
- \( \mathbb{P}^{0}, T_{N} \) -forward measure
- \( \mathbb{P}^{0}, T_{N-1} \) -forward measure
- \( \mathbb{P}^{0}, T_{j+1} \) -forward measure
- \( \mathbb{P}^{0}, T_{j} \) -forward measure
- \( \mathbb{P}^{0}, T_{1} \) -forward measure

**Foreign Market**
- \( \mathbb{P}^{i}, T^{*} \) -forward measure
- \( \mathbb{P}^{i}, T_{N} \) -forward measure
- \( \mathbb{P}^{i}, T_{N-1} \) -forward measure
- \( \mathbb{P}^{i}, T_{j+1} \) -forward measure
- \( \mathbb{P}^{i}, T_{j} \) -forward measure
- \( \mathbb{P}^{i}, T_{1} \) -forward measure

\[ F_{X_{i}}(\cdot, T^{*}) \]
\[ F_{X_{i}}(\cdot, T_{j}) \]
\[ F_{B}(\cdot, T_{j}, T_{j+1}) \]
\[ F_{B_{i}}(\cdot, T_{j}, T_{j+1}) \]

Relationship between domestic and foreign fixed income markets in a discrete-tenor framework.
Relationship between the domestic and the foreign market

The forward exchange rates in the \( i \)-th foreign market are related by

\[
F_{X^i}(t, T_j) = F_{X^i}(t, T_{j+1}) \frac{F_{B^i}(t, T_j, T_{j+1})}{F_{B^0}(t, T_j, T_{j+1})}
\]

From this one gets the dynamics of \( F_{X^i}(t, T_j) \)

\[
\frac{dF_{X^i}(t, T_j)}{dF_{X^i}(t-, T_j)} = \zeta^i(t, T_j, T_{j+1}) dW^{0,T_j}_t + \int_{\mathbb{R}^d} (\zeta^i(t, x, T_j, T_{j+1}) - 1)(\mu - \nu_{0,T_j})(dt, dx)
\]

where the coefficients are given recursively

\[
\zeta^i(t, T_j, T_{j+1}) = \alpha^i(t, T_j, T_{j+1}) - \alpha^0(t, T_j, T_{j+1}) + \zeta^i(t, T_{j+1}, T_{j+2})
\]

\[
\bar{\zeta}^i(t, x, T_j, T_{j+1}) = \frac{\beta^i(t, x, T_j, T_{j+1})}{\beta^0(t, x, T_j, T_{j+1})} \bar{\zeta}^i(t, x, T_{j+1}, T_{j+2})
\]
Pricing cross-currency derivatives

(1)

Foreign forward caps and floors

\[ \delta X[L^i(T_{j-1}, T_{j-1}) - K^i]^+ \]

Time-0-value of a foreign \( T_N \)-maturity cap

\[
FC^i(0, T_N) = \delta \sum_{j=1}^{N+1} B^i(0, T_j) \mathbb{E}_{\mathbb{P}^i,T_j} \left[ \left( L^i(T_{j-1}, T_{j-1}) - K^i \right)^+ \right]
\]

Alternatively if we define \( \tilde{K}^i = 1 + \delta K^i \) (forward process approach)

\[
FC^i(0, T_N) = \sum_{j=1}^{N+1} B^i(0, T_j) \mathbb{E}_{\mathbb{P}^i,T_j} \left[ \left( 1 + \delta L^i(T_{j-1}, T_{j-1}) - \tilde{K}^i \right)^+ \right],
\]

\[= \sum_{j=1}^{N+1} C^i(0, T_j, \tilde{K}^i) \]
Pricing cross-currency derivatives

(2)

Numerical evaluation of the cap price

Define

\[ X^i_{T_{j-1}}(t) = \int_0^t \chi^i(s, T_{j-1}) \, dL^i_s, \]

where \( \chi^i(s, T_{j-1}) \) is the characteristic function of the log-returns, and let \( \chi^{i,T_{j-1}}(z) \) be its characteristic function, then via a convolution representation

\[
C^i(0, T_j, \tilde{K}^i) = B^i(0, T_j)\tilde{K}^i \frac{\exp(\tilde{\xi}^i R)}{2\pi} \int_{-\infty}^{\infty} \frac{\exp(iu\tilde{\xi}^i)}{(R + iu)(1 + R + iu)} \, du
\]

where \( \tilde{\xi}^i = \ln(\tilde{K}^i) - \ln(1 + \delta L^i(0, T_{j-1})) \) and \( R \) is s.t. \( \chi^{i,T_{j-1}}(iR) < \infty \).
Pricing cross-currency derivates

Cross-currency swaps

Floating-for-floating cross-currency \((i; \ell; 0)\) swap

Libor rate \(L^i(T_{j-1}, T_{j-1})\) of currency \(i\) is received at each date \(T_j\)

Libor rate \(L^\ell(T_{j-1}, T_{j-1})\) of currency \(\ell\) is paid

Payments are made in units of the domestic currency

Thus the cashflow at time point \(T_j\) is (notional = 1)

\[
\delta(L^i(T_{j-1}, T_{j-1}) - L^\ell(T_{j-1}, T_{j-1}))
\]
Pricing cross-currency derivates

The time-0-value of a floating-for-floating \((i; \ell; 0)\) cross-currency forward swap in units of the domestic currency is

\[
CCFS_{[i;\ell;0]}(0) = B^0(0, T_J) \left( \sum_{j=1}^{N+1} \frac{B^i(0, T_{j-1})}{B^i(0, T_J)} \exp(D^i(0, T_{j-1}, T_J)) \right. \\
\left. - \sum_{j=1}^{N+1} \frac{B^\ell(0, T_{j-1})}{B^\ell(0, T_J)} \exp(D^\ell(0, T_{j-1}, T_J)) \right)
\]

where

\[
D^i(0, T_{j-1}, T_J) = - \int_0^{T_{j-1}} \lambda^i(s, T_{j-1})^\top c_s \zeta^i(s, T_J, T_{j+1}) \, ds \\
- \int_0^{T_{j-1}} \int_{\mathbb{R}^d} \left( \exp \left( \lambda^i(s, T_{j-1})^\top x \right) - 1 \right) \left( \zeta^i(s, x, T_J, T_{j+1}) - 1 \right) \nu_{0,T_J}(ds, dx)
\]
Pricing cross-currency derivates

(5)

A quanto caplet with strike $K^i$, which expires at time $T_{j-1}$, pays at time $T_j$

$$QCpl^i(T_j, T_j, K^i) = \delta \bar{X}^i(L^i(T_{j-1}, T_{j-1}) - K^i)^+$$

where $\bar{X}^i$ is the preassigned foreign exchange rate

Time-0-value

$$QCpl^i(0, T_j, K^i) = B^0(0, T_j)\mathbb{E}_{\mathbb{P}_0, T_j}[\delta \bar{X}^i(L^i(T_{j-1}, T_{j-1}) - K^i)^+]$$

$$= B^0(0, T_j)\bar{X}^i\mathbb{E}_{\mathbb{P}_0, T_j}[(1 + \delta L^i(T_{j-1}, T_{j-1}) - (1 + \delta K^i))^+]$$

(forward process approach)
Pricing cross-currency derivates

(6)

Numerical evaluation of quanto caplets. Write

\[ 1 + \delta L^i(T_{j-1}, T_j) = (1 + \delta L^i(0, T_{j-1})) \exp \left( \int_0^{T_{j-1}} \lambda^i(s, T_{j-1}) \, dL^i_s(T_j) \right) \]

\[ = (1 + \delta L^i(0, T_{j-1})) \exp \left( \lambda^i(0, T_{j-1}, T_j) \right) \]

\[ = (1 + \delta L^i(0, T_{j-1})) \exp \left( \lambda^i(0, T_{j-1}, T_j) + D^i(0, T_{j-1}, T_j) \right) \]

\text{assume density } \varrho \text{ \ non-random}

then for \( \nu(x) = (e^{-x} - 1)^+ \)

\[ QCpl^i(0, T_j, K^i) = B^0(0, T_j) \overline{X}^i(1 + \delta K^i)(\nu * \varrho)(\xi_j) \]

Finally we get

\[ QCpl^i(0, T_j, K^i) = B^0(0, T_j) \overline{X}^i(1 + \delta K^i) \]

\[ \cdot \frac{\exp(\xi_j R)}{2\pi} \int_{-\infty}^{\infty} \exp(iu\xi_j) \frac{\chi^{\lambda^i(0, T_{j-1})}(iR - u)}{(R + iu)(R + 1 + iu)} \, du \]
Absolute errors of EUR caplet calibration
Absolute errors of USD caplet calibration

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<td>1.0%</td>
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References

References (cont.)