The Hilbert Transform and the maximal (Hardy-Littlewwod) operator along variable families of non-flat curves.

Abstract: In this talk we will investigate the following

Main Problem: *Let* $\Gamma_{(x,y)} = (t,\, \gamma(x,y,t))$ *be a* *variable* *curve in the plane, where here* $t \in \mathbb{R}$ *and* $(x,y) \in \mathbb{R}^2$ *while*

$$\gamma_{(x,y)}(\cdot) := \gamma(x,y,\cdot) : \mathbb{R} \to \mathbb{R}$$

is a "suitable" real function. Under what conditions on the curve $\Gamma_{(x,y)}$ *- (our main target: minimal regularity in* x *and* y*) - do we have that the* Hilbert transform along curve Γ *defined by*

$$H_\Gamma f(x,y) := p.v. \int_{\mathbb{R}} f(x-t,\, y+\gamma(x,y,t))\, \frac{dt}{t}\,,$$

and the maximal (Hardy-Littlewood) operator along curve Γ *defined by*

$$M_\Gamma f(x,y) := \sup_{\epsilon>0} \frac{1}{2\epsilon} \int_{-\epsilon}^{\epsilon} |f(x-t,\, y+\gamma(x,y,t))|\, dt\,,$$

are bounded operators from $L^p(\mathbb{R}^2)$ *to* $L^p(\mathbb{R}^2)$ *for* $1 < p < \infty$*?*

We will insist on the history and motivation for this problem from

- the PDE perspective - the study of constant coefficient parabolic differential operators and
- the Harmonic Analysis perspective - connections with singular maximal operators and the conjecture of A. Zygmund on maximal integrals along (Lipschitz) vector fields.

If the time allows we will very briefly outline some of the methods we used in obtaining our most recent results.