ALMOST SURE CONVERGENCE OF THE KACZMARZ ALGORITHM WITH RANDOM MEASUREMENTS

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Abstract: The Kaczmarz algorithm is an iterative method for reconstructing a signal $x \in \mathbb{R}^d$ from an overcomplete collection of linear measurements $y_n = \langle x, \varphi_n \rangle$, $n \geq 1$. We prove quantitative bounds on the rate of almost sure exponential convergence in the Kaczmarz algorithm for suitable classes of random measurement vectors $\{\varphi_n\}_{n=1}^{\infty} \subset \mathbb{R}^d$. Refined convergence results are given for the special case when each φ_n has i.i.d. Gaussian entries and, more generally, when each $\varphi_n / \|\varphi_n\|$ is uniformly distributed on $\mathbb{S}^{d-1}$. Interestingly, the work on finding the best convergence rate among all the probability measures on the sphere is closely related to logarithmic potential theory. The conclusion is that the best convergence rate occurs when $\varphi_n / \|\varphi_n\|$ is uniformly distributed on $\mathbb{S}^{d-1}$.