Scattering Transform for Art Investigation

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Joint Work with Roberto Leonarduzzi and Haixia Liu

Celebrating the 80th Birthday of John Benedetto

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Content

Contents

Linear Classification Problem

Notation

- **Feature vector:** $f \in \mathbb{R}^p$
- **Output:** $y \in \{1, 2\}$
- **Training data:** $\mathcal{T} = \{f_i, y_i = y(f_i)\}_{i=1,\ldots,N}$

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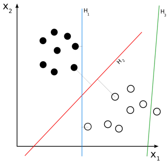
General Form

$$\hat{y}(f) = \begin{cases} 1 & \text{if } \hat{w}^T f > \hat{t} \\ 2 & \text{otherwise,} \end{cases}$$



Source: Wikipedia

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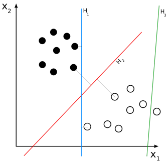


Source: Wikipedia

Learning Algorithm

$$(\hat{w}, \hat{t}) \in \arg\min \mathcal{L}(y_i, \hat{y}(f_i)) \quad (+\ \gamma \|w\|_1)$$

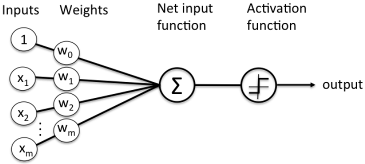
Several choices of loss $\mathcal{L}$: LDA, SVM, perceptron

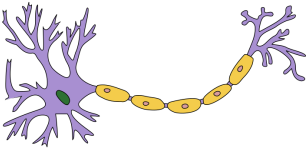
Linear Perceptron

- Simple linear classifier

$$y = \operatorname{sgn}\left(\sum_i w_i x_i\right) = GWx$$

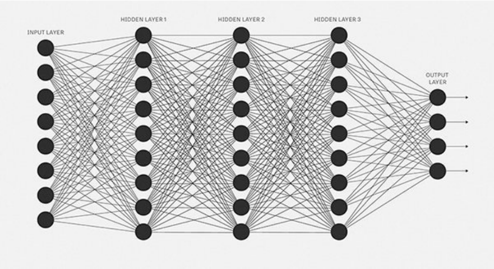
- W, G: linear, nonlinear operators
- Weight learning: gradient descent
- Simplified model of real neurons





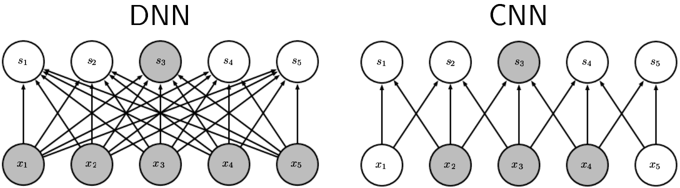
Deep Neural Networks (DNNs)

- Large array of perceptrons in many layers



- Output: $\hat{y} = \ G_M W_M \ \cdots G_2 W_2 \ G_1 W_1 x$
- Deep: $M \gg 2$
- Linear operators $\{W_m\}$ learned from data
  $\longrightarrow$ Error back-propagation algorithm

Convolutional Neural Networks (CNNs)



Source [Goodfellow, et al., 2016]

- "Parameter sharing" and "sparse activations"
- Operators W_m are convolutions
- Easier to learn (less weights)
- Successfully used in image processing tasks

[LeCun et al., 1989][Ciresan et al., 2012][Krizhevsky et al., 2012]

Contents

Scattering Transform

[Mallat, 2012]

- Structure of Convolutional Neural Network
- Replace linear filters by wavelets
- Use modulus as nonlinearity

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Notation

- $\lambda = \lambda(j,\theta) = a^{-j}\theta,\ j \in \mathbb{Z},\ \theta \in R \subset SO(d)$
- $p = (\lambda_1, \lambda_2, \ldots, \lambda_M)$
- $\psi_\lambda(u) = 2^{-dj}\psi(\lambda_i x)$
- $\phi_J(u) = 2^{-dJ}\phi(2^{-J}x)$

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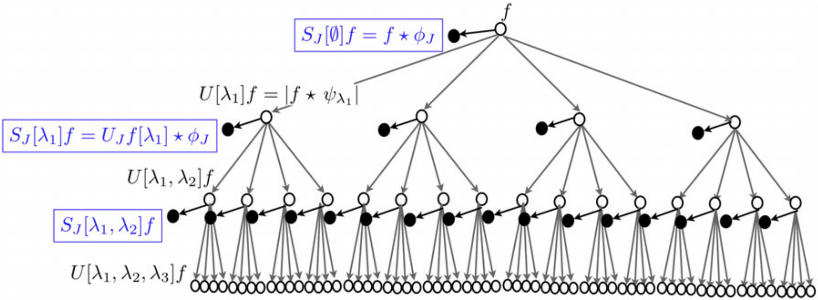
Scattering coefficients

$$S_m[p]X(u) = |\,|\,||X \star \psi_{\lambda_1}| \star \psi_{\lambda_2}| \star \cdots \star | \star \psi_{\lambda_m}| \star \phi_J(u)$$

$$S_m[P]X = \left(S_m[p]X\right)_{p \in P}$$

Illustration



Source: [Mallat, 2012]

Properties

Stability

$$\forall X, Y \in L^2(\mathbb{R}^d), \quad \|S[P]X - S[P]Y\| \leq \|X - Y\|$$

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Translation invariance

Let $T_c X(u) = X(u - c)$. Then,

$$\forall X \in L^2(\mathbb{R}^d), \forall c \in \mathbb{R}^d, \qquad S[P]T_c X = S[P]X, \quad \text{when } J \to \infty$$

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Stability to deformations

Let $D_\tau X(u) = X(u - \tau(u))$ with $\|\nabla \tau\|_\infty \leq \frac{1}{2}$. Then,

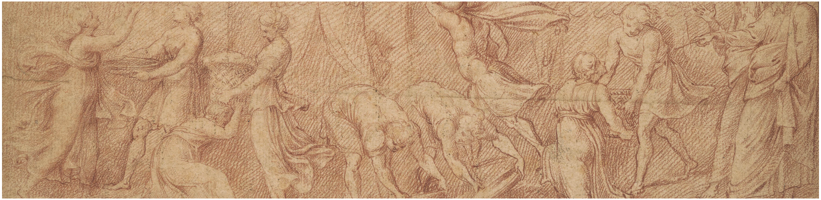
$$\forall X \in L^2(\mathbb{R}^d), \forall \tau \in C^2(\mathbb{R}^d), \qquad \|S[P]X - S[P]D_\tau X\| \leq C\|X\|\|\nabla \tau\|_\infty$$

[Mallat2012]

Contents

Art Authentication: Is It a Raphael?

In 2013 I have received a request from a collector Edward Rosser in Boston, asking me whether I could tell the following drawing was a genuine Raphael.



Art Authentication Problem

- Detect art objects from forgeries or imitations
- Style vs content
- Database 1
  - 64 van Gogh paintings (several periods)
  - 15 forgeries or contemporaries in same style
  - Size: 1452×388 px to 5614×7381 px
- Database 2
  - 21 drawings by Raphael
  - 9 imitations
  - Size: 2188×3312 to 6330×4288 pixels

Sample Images

van Gogh (VG)

non van Gogh (NVG)

Raphael (RA)

non Raphael (NRA)

State of the Art

Van Gogh dataset: Features and classifiers

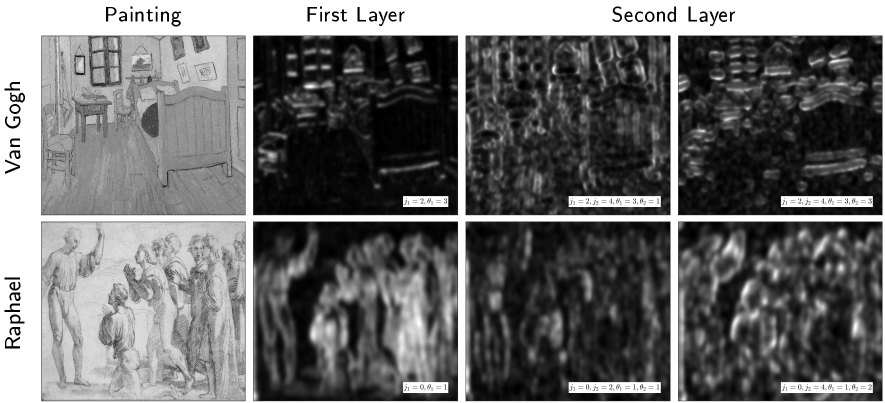
- Wavelets, custom frames, EMD
- SVM, clustering, Hidden Markov Models
- Accuracy $< 90\%$
- Single layer of features

[Berezhnoy et al. 2007], [Johnson et al., 2008], [Qi et al., 2013], [Liu & Chan, 2016]

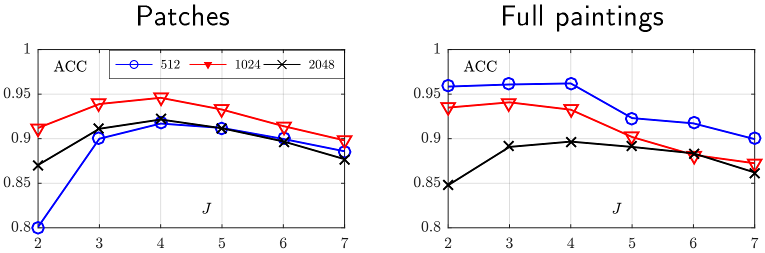
Analysis Setup

- Preprocessing: grayscale, $[0, 1]$ double-precision
- Automatic removal of canvas edges (max 100 px)
- Morlet wavelets, $a = 2$, 8 rotations
- $J = 3, 4, \ldots, 7$
- Analysis by patches: 512×512, 1024×1024 and 2048×2048
- 5-fold stratified cross-validation
- Linear classifiers:
  - PCA, LDA, SVM
  - **Sparse versions:** SSVM, SLDA $\longrightarrow$ ℓ_1 regularization

Example: Scattering Coefficients

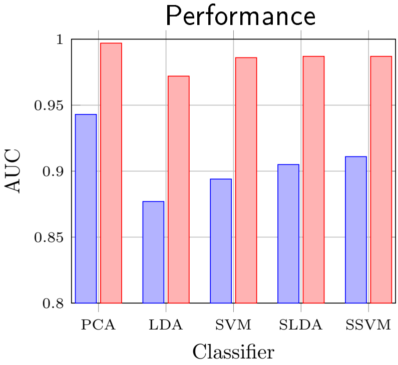


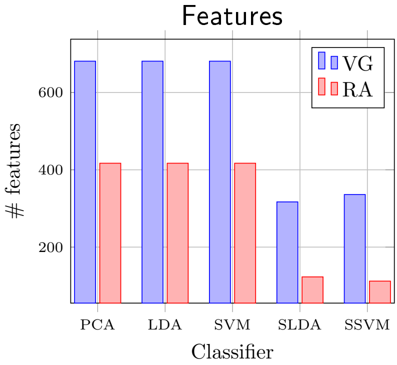
Influence of Patch Size and Averaging Scale



- Select 512×512 and $J = 4$.
- Fine-scale details preferred

Performance: Individual Patches

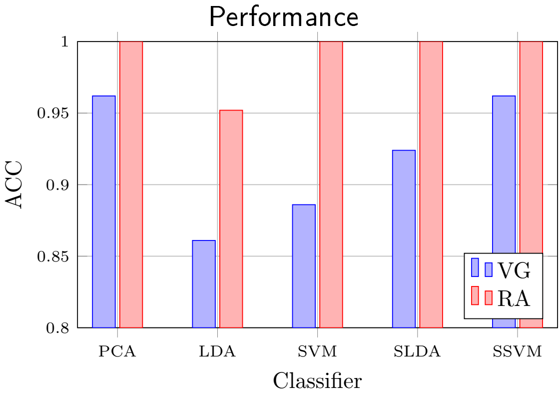




- Simple is better (PCA)
- Sparse is better (SLDA/SSVM)
- Raphael easier than van Gogh

Performance: Full Paintings

- Majority votes from patch decisions



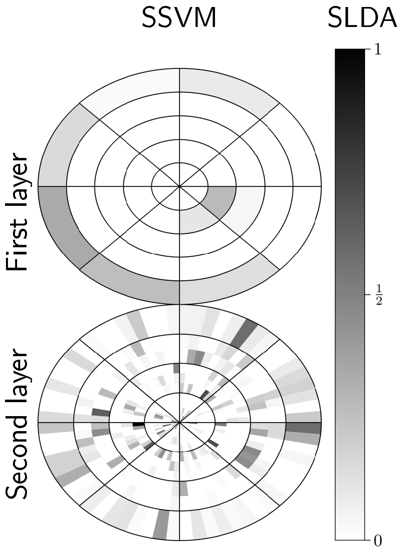
- SSVM: good performance & few features

Performance: Comparison with State of the Art

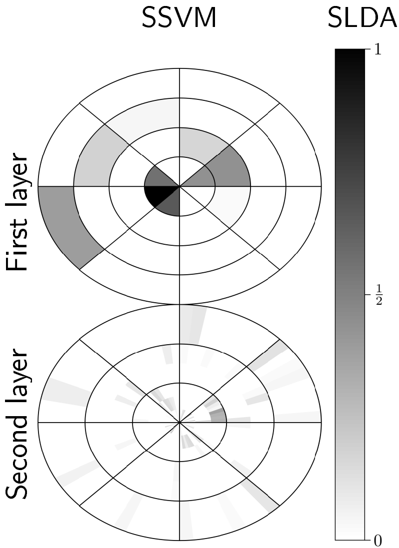
- Similar Van Gogh database

Reference	ACC	Data size (VG+NVG)	Validation
[Liu & Chan, 2016]	0.88	$64 + 15$	LOO
[Qi et al., 2013]	0.85	$65 + 15$	LOO
[Johnson et al., 2008]	0.84	$64 + 12$	LOO
Our results	0.96	$64 + 15$	5-CV

Feature Selection: van Gogh



Feature Selection: Raphael



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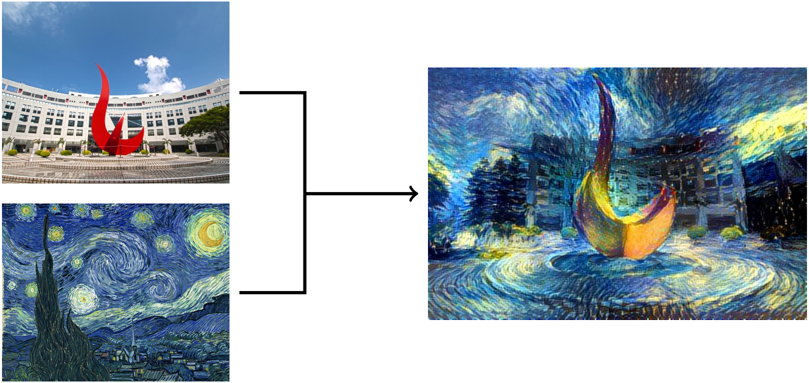
Neural Style Transfer



How Does It Work?

- Pretrained convolutional neural network
- **Coefficient matrix:** $F_m(X) \in \mathbb{R}^{N_{filt} \times N_{pixels}}$
- **Correlation matrix:** $G_m(X) = \frac{1}{N_{pixels}} F_m(X) \big(F_m(X)\big)^T$

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$$\mathcal{L}_{total}(X) = \alpha \mathcal{L}_{content}(X; m_0) + \beta \sum_m w_m \mathcal{L}_{style}(X; m)$$

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- **Image synthesis:**

$$X \in \arg\min_X (\mathcal{L}_{total})$$

[Gatys et al., 2015]

Style Transfer: Scattering

Ideas

1. Simple manipulation of coefficients
2. Scattering can be inverted

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2. Scattering can be inverted
    - Phase retrieval + Pseudoinverse
    - Seems easy...
    - ...it's not
    - Limitation: current phase retrieval algorithms

Covariance Change

Notation

- **Coefficient matrix:** $F_m \in \mathbb{R}^{N_{filt} \times N_{pixels}}$
- $\Sigma_m = cov(F_m) = \frac{1}{N_{pixels}} F_m F_m^T = U \Lambda U^T$

Procedure

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$$F_m^{white} = \Lambda^{-1/2} U^T F_m$$

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$$\Sigma_m^{new} = \alpha \Sigma_m^{cont} + (1 - \alpha) \Sigma_m^{style}$$
3. Transfer style
$$F_m^{new} = \left(\alpha \Sigma_m^{cont} + (1 - \alpha) \Sigma_m^{style} \right)^{1/2} \Lambda^{-1/2} U^T F_m$$

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4. Invert F_m^{white}

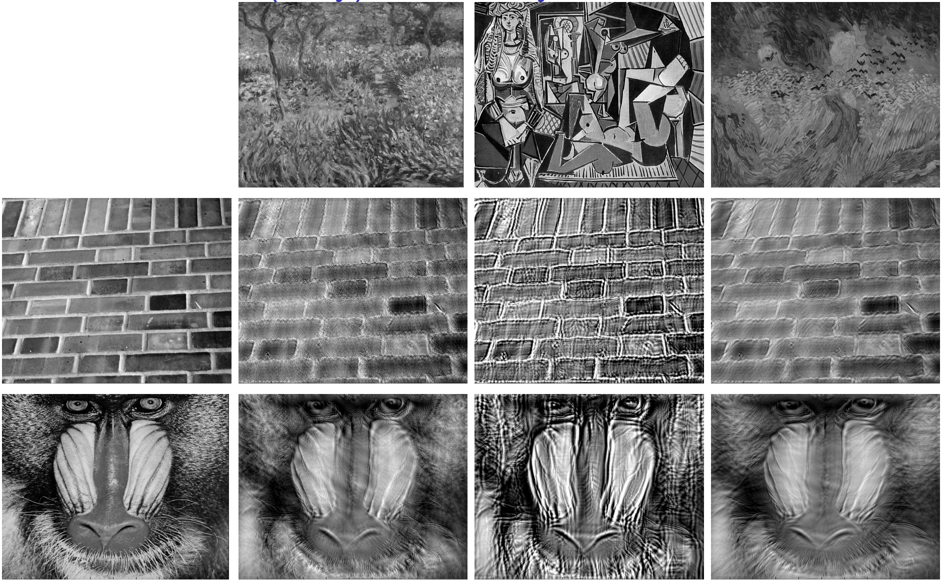
Phase Retrieval

- Gerchberg-Saxton algorithm
    - Recover x such that $|Ax| = b$
    - Input: $y^{(1)} \in \mathbb{C}$ such that $|y^{(1)}| = b$
    - Iteration:

$$y_i^{(k+1)} = b_i \frac{(AA^\dagger y^{(k)})_i}{|(AA^\dagger y^{(k)})_i|}$$

    - Not guaranteed to converge to solution
    - Low computational complexity

(Very) Preliminary Results




Happy 80th Birthday John!