Frames for Psychoacoustics
Erblet transform and perceptual sparsity

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joint work with T. Necciari, B. Laback, N. Holighaus, D. Stoeva, ...

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Acoustics Research Institute (ARI)

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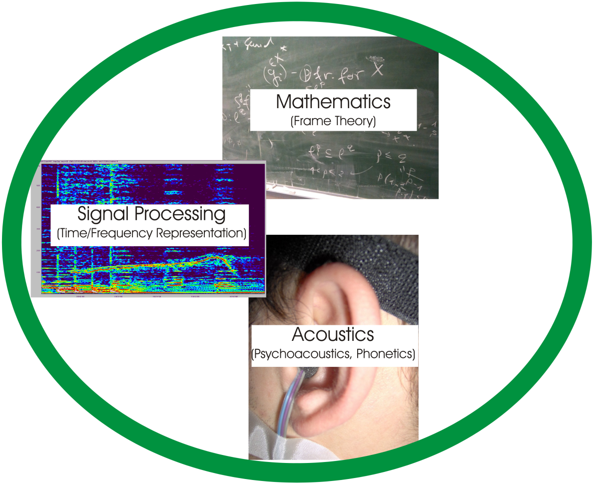
Interdisciplinary research in acoustics, integrating acoustic phonetics, psychoacoustics and computational physics, based on a solid mathematical background.

Excellence through Synergy

ARI

Advantage of a strong mathematical background

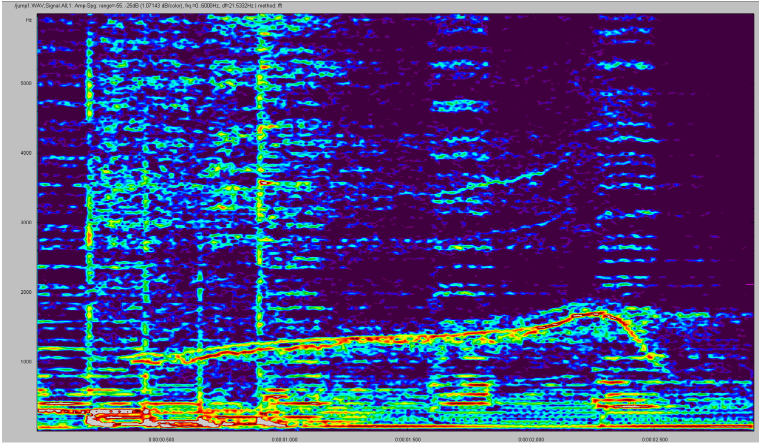


- Application-oriented mathematics
- True inter-disciplinarity
- Synergy
- Novel methods

Overview:

Signal Representations: Time-Frequency Analysis and Frames

Spectrogram



Short Time Fourier Transformation (STFT)

Definition (see e.g. [Gröchenig, 2001])

Let $f,g \neq 0$ in $L^2\left(\mathbb{R}^d\right)$, then we call

$$\mathcal{V}_g f(\tau,\omega) = \int\limits_{\mathbb{R}^d} f(x)\overline{g(x-\tau)}e^{-2\pi i \omega x}dx$$

the Short Time Fourier Transformation (STFT) of the signal f with the window g.

Sampled Version is the Gabor transform:

$$f \mapsto \mathcal{V}_g(f)(a\cdot k, b\cdot l) = \langle f, g_{k,l}\rangle \text{, where } g_{k,l}(t) = g(t-ka)e^{i2\pi lbt}.$$

When is perfect reconstruction possible?

Frames

Definition

The (countable) sequence $\Psi = (\psi_k | k \in K)$ is called a frame for the Hilbert space $\mathcal{H}$ if constants $A > 0$ and $B < \infty$ exist such that

$$A \cdot \|f\|_{\mathcal{H}}^2 \leq \sum_k |\langle f, \psi_k \rangle|^2 \leq B \cdot \|f\|_{\mathcal{H}}^2 \, , \; \forall \; f \in \mathcal{H}.$$

[Duffin and Schaeffer, 1952, Daubechies et al., 1986]

Beautiful abstract mathematical setting:

- Frames = generalization of bases; can be overcomplete, allowing redundant representations. Redundancy
- Active field of research in mathematics!

Interesting for applications:

- **Much more freedom.** Finding and constructing frames can be easier and faster.
  Some advantageous side constraints can **only** be fulfilled for frames.
- Perfect reconstruction is guaranteed with the 'canonical dual frame' $\tilde{\psi}_k = S^{-1}\psi_k$

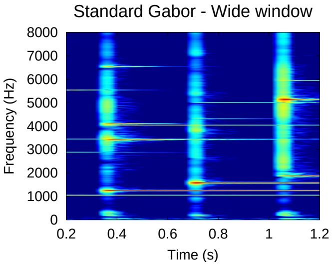
$$f = \sum_k < f, \psi_k > \tilde{\psi}_k = \sum_k < f, \tilde{\psi}_k > \psi_k,$$

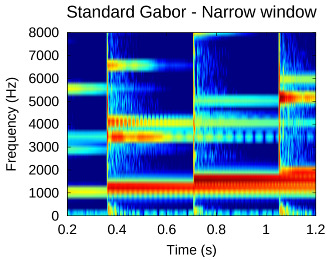
where S is the frame operator $Sf = \sum_k < f, \psi_k > \psi_k$.

Frame Theory:

Non-stationary Gabor transform

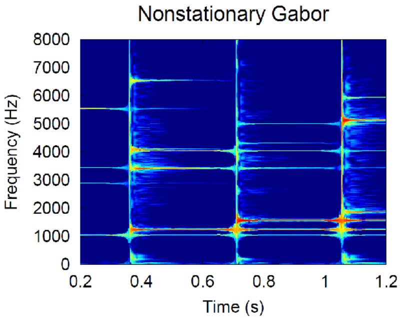
Limitations of Standard Gabor analysis: Quality of representation highly depends on window choice, but optimal window choice is different for different signal components





Non-stationary Gabor Transform

Our proposition [Balazs et al., 2011]: simple extension to reduce this limitation by using windows evolving over time.



Non-stationary Gabor Transform

Given a sequence of windows $(g_n)_{n \in \mathbb{Z}}$ of $L^2(\mathbb{R})$ and sequences of real numbers $(a_n)_{n \in \mathbb{Z}}$ and $(b_n)_{n \in \mathbb{Z}}$, the non-stationary Gabor transform (NSGT) elements are defined, for $(m, n) \in \mathbb{Z}^2$, by:

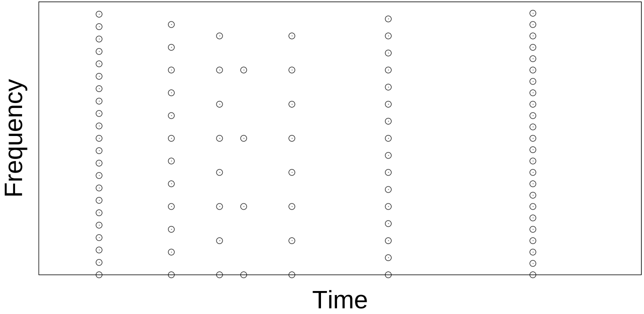
$$g_{m,n}(t) = g_n(t - na_n)e^{i2\pi m b_n t}.$$

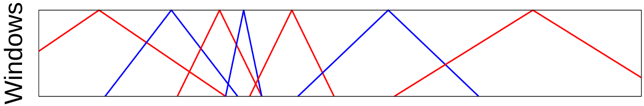
Regular structure in frequency allows FFT implementation.

An analogue construction in the frequency domain allows easy implementation of, e.g. wavelet frames; an invertible CQT [Velasco et al., 2011].

Non-stationary Gabor Transform

Sampling grid example:





Non-stationary Gabor Transform

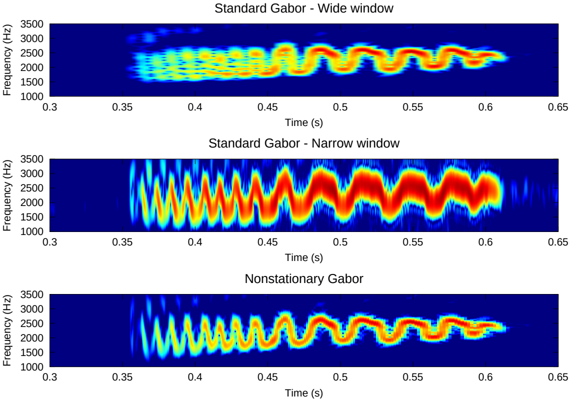
Frame theory allows perfect reconstruction. Particularly efficient in the 'painless' case:

Theorem

For every $n \in \mathbb{Z}$, let the function $g_n \in L^2(\mathbb{R})$ be compactly supported with $\operatorname{supp}(g_n) \subseteq [c_n, d_n]$ such that $d_n - c_n \leq \frac{1}{b_n}$. The system of functions $g_{m,n}$ forms a frame for $L^2(\mathbb{R})$ if and only if there exists $A > 0$ and $B < \infty$, such that $A \leq \sum_n \frac{1}{b_n} |g_n(t - na_n)|^2 \leq B$. In this case, the canonical dual frame has the same structure and is given by:

$$\tilde{g}_{m,n}(t) = \frac{g_n(t)}{\sum_k \frac{1}{b_k} |g_k(t - ka_k)|^2} e^{2\pi i m b_n t}. \tag{1}$$

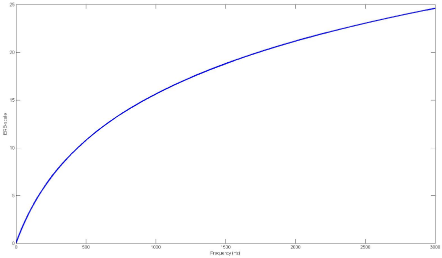
Bird vocalization example:



For an overview of adapted and adaptive time-frequency representations, see [Balazs et al., 2013].

ERBlets

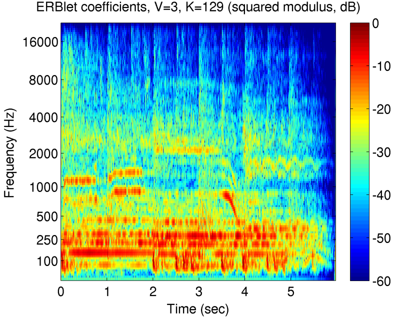
Non-stationary Gabor transform adapted to human auditory perception [Necciari et al., 2013]:



ERB-scale

ERBlets

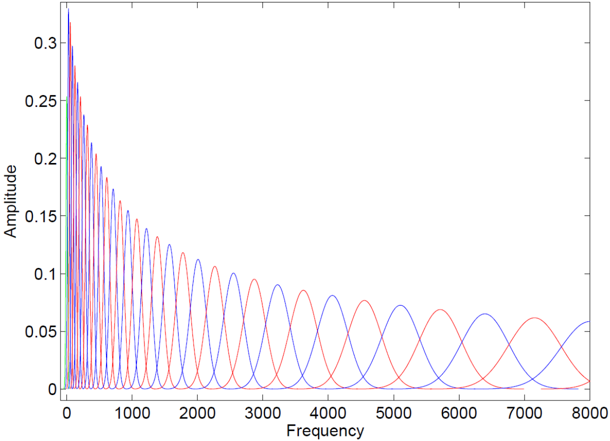
Non-stationary Gabor transform adapted to human auditory perception [Necciari et al., 2013]:



Relative reconstruction error: $< 10^{-15}$.
Implementation in LTFAT [Soendergaard et al., 2012].

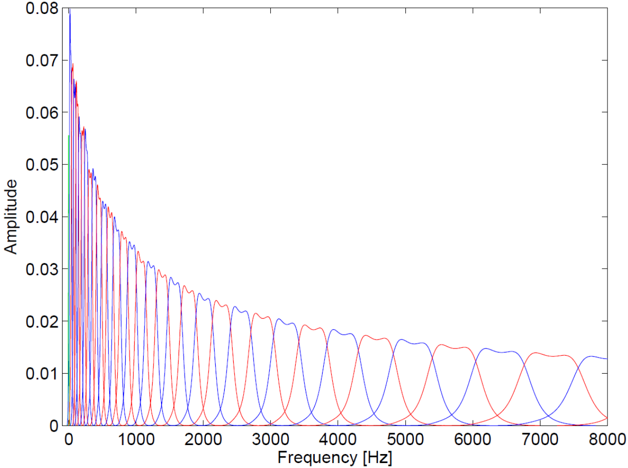
ERBlets

Filterbank:



ERBlets

Dual Filterbank:



What is a

Frame Multiplier:

Analysis

$\Downarrow$

Multiplication

$\Downarrow$

Synthesis

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Synthesis

What is a

Frame Multiplier:

Analysis

$$\Downarrow$$

Multiplication

$$\Downarrow$$

Synthesis

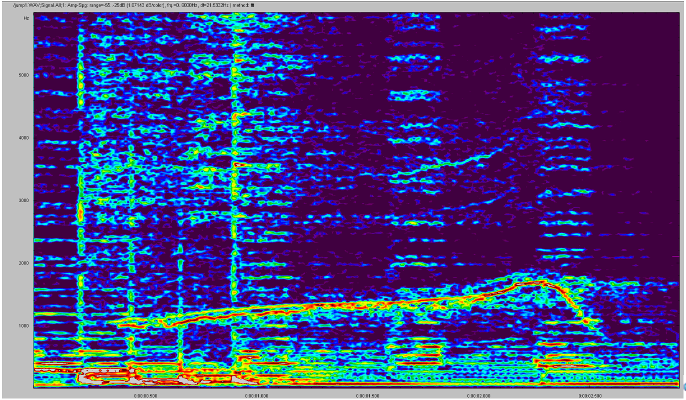
Multipliers

Those are operators, that are of utmost importance in

- Mathematics, where they are used for the diagonalization of operators [Schatten, 1960].
- Physics, where they are a link between classical and quantum mechanics, so called quantization operators [Ali et al., 2000].
- Signal Processing, where they are a particular way to implement time-variant filters [Matz and Hlawatsch, 2002].
- Acoustics, where those time-frequency filters are used in several fields, for example in Computational Auditory Scene Analysis [Wang and Brown, 2006].

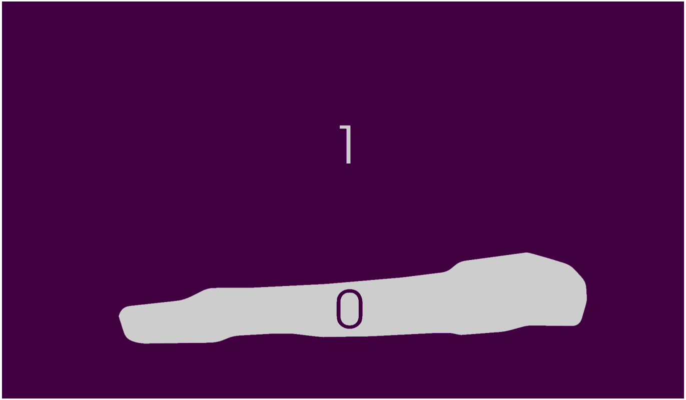
Example for a Multiplier

Original audio file:



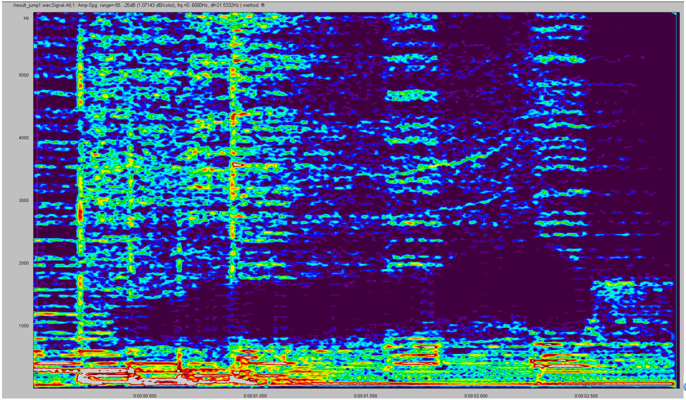
Example for a Multiplier

Symbol:



Example for a Multiplier

Result of Gabor Multiplier.



Frame Multipliers: Definition

Definition ([Balazs, 2007])

Let $(\psi_k)_{k\in K}$, $(\phi_k)_{k\in K}$ be frames for the Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$. Define the frame multiplier $\mathbf{M}_{(m_k),(\phi_k),(\psi_k)} : \mathcal{H}_1 \to \mathcal{H}_2$ as the operator

$$\mathbf{M}_{(m_k),(\phi_k),(\psi_k)} f = \sum_k m_k \langle f, \psi_k \rangle \phi_k,$$

where $m = (m_k)$ is called the symbol.

Generalization of Gabor multipliers [Feichtinger and Nowak, 2003].

We have invested quite some afford into the abstract setting, in particular investigating invertible multipliers, see e.g. [Stoeva and Balazs, 2012].

Theorem (B., Stoeva; submitted)

Let Φ and Ψ be frames for $\mathcal{H}$, and let m be semi-normalized. Let $\mathbf{M}_{m,\Phi,\Psi}$ be invertible. Then there exist a dual frame $\Phi^\dagger$ of Φ and a dual frame $\Psi^\dagger$ of Ψ, so that for any dual frame Φ^d of Φ and any dual frame Ψ^d of Ψ we have

$$M_{m,\Phi,\Psi}^{-1} = M_{1/m,\Psi^\dagger,\Phi^d} = M_{1/m,\Psi^d,\Phi^\dagger}. \tag{2}$$

The frames $\Psi^\dagger$ are uniquely determined.

Applications in Acoustics: **Perceptual Sparsity by Irrelevance**

MP3-Player

MP3:

- encoding / decoding scheme
- MPEG1/MPEG2 (Layer 3)
- signal processing
- psychoacoustical masking model

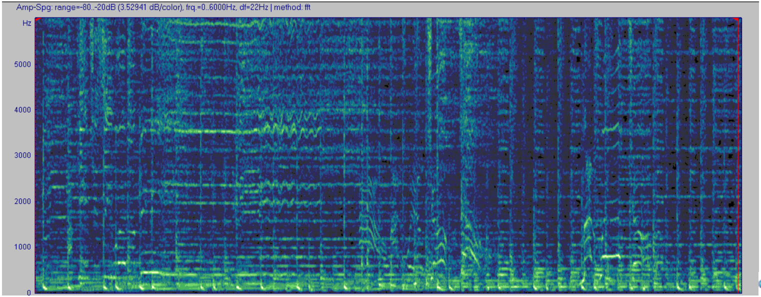
Masking:
presence of one stimulus, the masker, decreases the detectability of another stimulus, the target.

Irrelevance Filter: searches (and deletes) perceptual irrelevant (masked, inaudible) data (in complex signals) using a masking model.

Masking:
presence of one stimulus, the masker, decreases the detectability of another stimulus, the target.

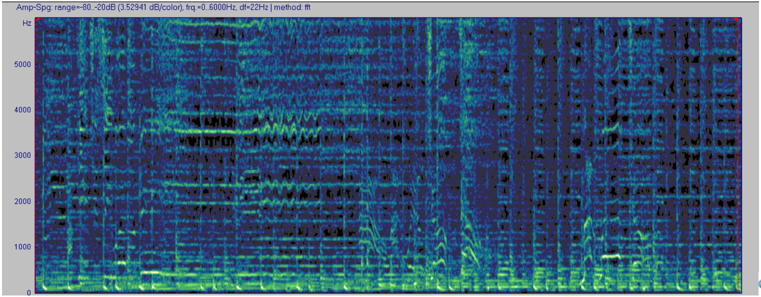
Irrelevance Filter: searches (and deletes) perceptual irrelevant (masked, inaudible) data (in complex signals) using a masking model.

Algorithm in STx:
Original audio file



Algorithm in STx:

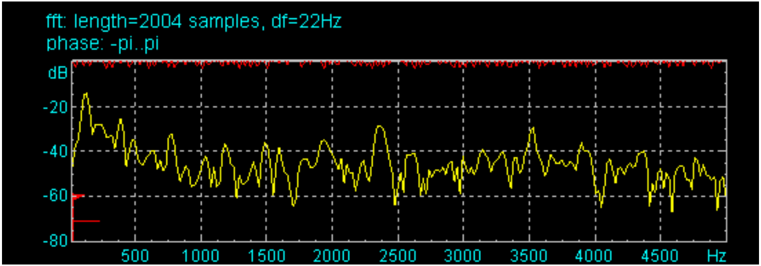
Filtered signal



Residual

Adaptive

Existing algorithm in STX: Original audio file (Spectrum)



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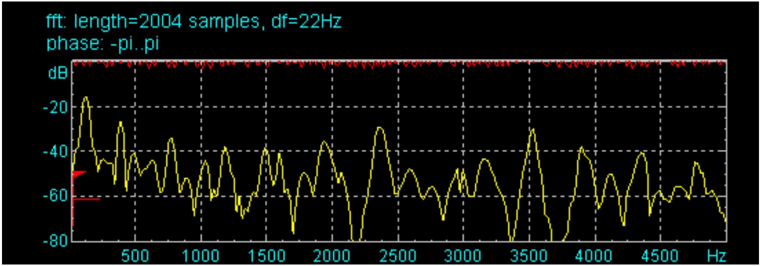
Irrelevance Filter

Existing algorithm in STx: Masked signal (Spectrum)



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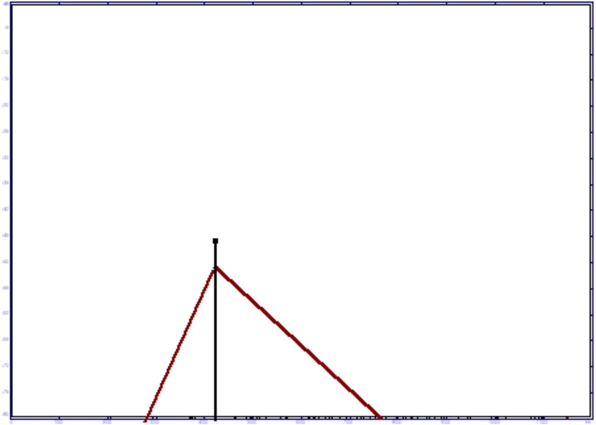
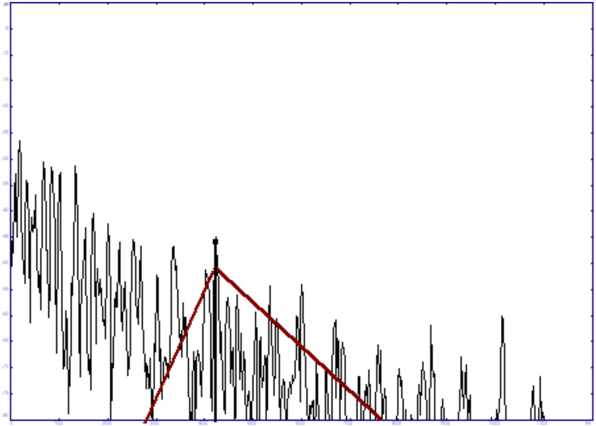
Psychoacoustic Masking : simultaneous masking I

Existing Model, using bark scale

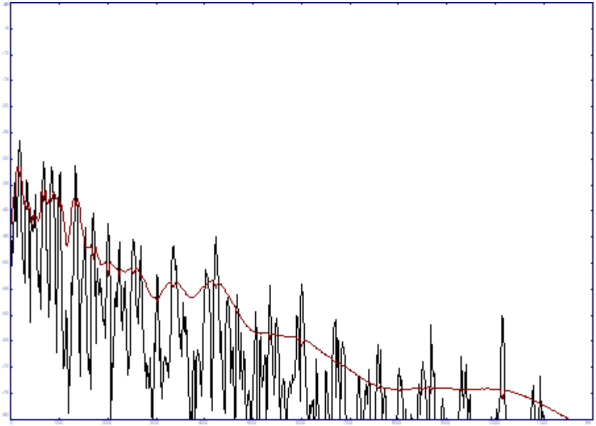


Existing Model, using bark scale



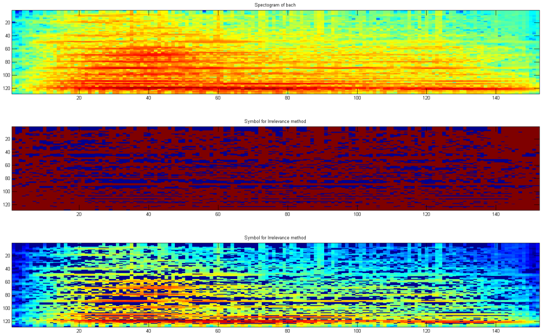
Psychoacoustic Masking : simultaneous masking I

Existing Model, using bark scale



Perceptual Sparsity by Irrelevance

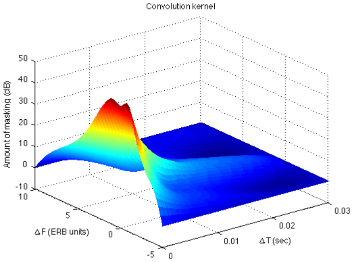
The irrelevance method calculates an adaptive threshold function for each spectra of a Gabor transform. This corresponds to an adaptive Gabor frame multiplier with coefficients in $\{0, 1\}$.



Perceptual Sparsity: Current and Future Work

Extend to True Time-Frequency Model using Multipliers:

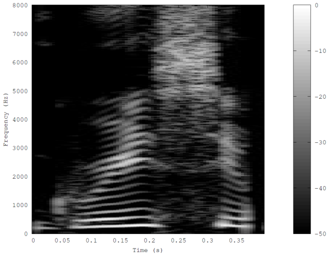
- Base it on ERBlets.
- Use new psychoacoustical data on time-frequency masking [Necciari et al., 2012].

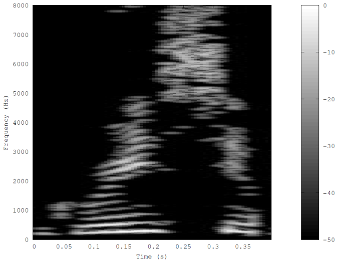


Use this for improved audio codec!

Perceptual Sparsity: Current and Future Work

Perceptual OMP:





OMP reduces to 400 atoms, masking removes another 73.

Frame theory is

- not only a very beautiful abstract setting,
- but also important for applications,
- in particular by linking it to human hearing!

Thank you for your attention!

- http://www.kfs.oeaw.ac.at
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Frames for Psychoacoustics

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Frame Theory

Multipliers

Perceptual Sparsity by Irrelevance

Conclusions

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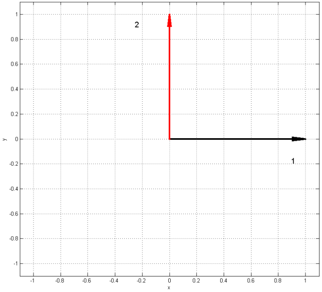
Orthonormal Basis (ONB)

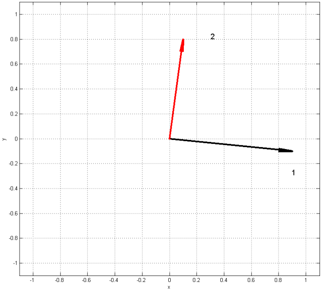
Standard aproach: orthonormal basis.



Problems:

- Perturbation
- Construction
- Error Robustness

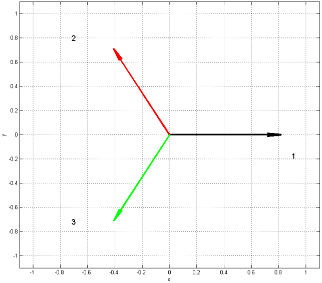
Riesz bases



Problems:

- ~~Perturbation~~
- Construction
- Error Robustness

Alternate approach: introduce redundancy.



Problems:

- ~~Perturbation~~
- ~~Construction~~
- ~~Error Robustness~~

Back