Compressive Sensing Super Resolution Camera

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in collaboration with

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Summary

- Compressive Sensing
- Camera Design Concept
- Recostruction Algorithms
- Experimental Results

Compressive Sensing

Suppose $x \in \mathbb{C}^N$ is K-sparse in a basis, or more generally, a frame $\mathbf{D}$, so that $\mathbf{x} = \mathbf{D}\alpha_0$, with $\| \alpha_0 \|_0 = \mathbf{K} \ll \mathbf{N}$, where $\| \alpha_0 \|_0$ returns the number of nonzero elements of α_0. In the case when $\mathbf{x}$ is compressible in $\mathbf{D}$, it can be well approximated by the best $\mathbf{K}$-term representation.

Consider an $\mathbf{M} \times \mathbf{N}$ measurement matrix Φ with $\mathbf{M} < \mathbf{N}$ and assume that $\mathbf{M}$ linear measurements are made such that $\mathbf{y} = \Phi\mathbf{x} = \Phi\mathbf{D}\alpha_0 = \Theta\alpha_0$. Having observed $\mathbf{y}$ and knowing the matrix Θ, the general problem is to recover α_0.

Compressive Sensing

Estimate

$$(P_0) \quad \arg\min_{\alpha_0'} \| \alpha_0' \|_0 \quad \text{subject to} \quad y = \Theta\alpha_0'.$$

Unfortunately, (P_0) is NP-hard and is computationally difficult to solve.

Relaxed Estimate

$$(P_1) \quad \arg\min_{\alpha_0'} \| \alpha_0' \|_1 \quad \text{subject to} \quad y = \Theta\alpha_0',$$

where $\|\alpha\|_1 = \sum_i |\alpha_i|$.

In the case when there are noisy observations of the following form

$$y = \Theta\alpha_0 + \eta$$

with $\|\eta\|_2 \leq \varepsilon$, Basis Pursuit De-Noising (BPDN) can be used to approximate the original image.

Relaxed Denoised Estimate

$$(P_1^\varepsilon) \quad \arg\min_{\alpha_0'} \lambda \parallel \alpha_0' \parallel_1 + \frac{1}{2}\|y - \Theta\alpha_0'\|_2^2.$$

Definition (Restricted Isometry Property)
For each integer $K = 1, 2, \ldots,$, define the isometry constant δ_K of a matrix Θ as the smallest number such that

$$(1-\delta_K)\|\alpha_0\|_2^2 \leq \|\Theta\alpha_0\|_2^2 \leq (1+\delta_K)\|\alpha_0\|_2^2$$

holds for all K-sparse vectors.

• α_0^* will denote the *best sparse approximation* one could obtain if one knew exactly the locations and amplitudes of the K-largest entries of α_0.
• $\alpha_0|_K$ will denote the vector α_0 with all but the K-largest entries set to zero.

We can now state the following result assuming the Θ obeys the *restricted isometry property*(RIP).

Theorem (Candés, 2008)

Assume that $\delta_{2K} < \sqrt{2} - 1$. Then the solution α_0^* to (P_1) obeys

$$\|\alpha_0^* - \alpha_0\|_1 \leq C_0 \|\alpha_0 - \alpha_0|_K\|_1 \;\&\; \|\alpha_0^* - \alpha_0\|_2 \leq C_0 K^{-1/2} \|\alpha_0 - \alpha_0|_K\|_1,$$

for a particular constant C_0. In particular, if α_0 is K-sparse, the recovery is exact.

Furthermore, if $\delta_{2K} < 1$, then (P_0) has a unique K-sparse solution, and if $\delta_{2K} < \sqrt{2} - 1$, the solution to (P_1) is that of (P_0).

Theorem (Candés, 2008)

Assume that $\delta_{2K} < \sqrt{2} - 1$. Then the solution α_0^* to (P_1^ε) obeys

$$\|\alpha_0^* - \alpha_0\| \leq C_0 K^{-1/2} \|\alpha_0 - \alpha_0|_K\|_1 + C_1 \varepsilon,$$

for some particularly small constants C_0 and C_1.

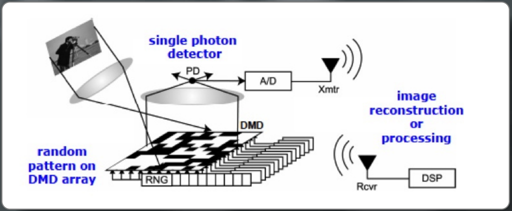
Compressive Sensing

Examples of $\Theta's$ that obey the RIP when $M = \mathcal{O}(K \log(N/K))$ occur when

- Φ contains random Gaussian elements
- Φ contains random binary elements
- Φ contains randomly selected Fourier samples

Our physical system will limit us to the case when Φ contains random binary elements.

The Rice Single Pixel Camera:

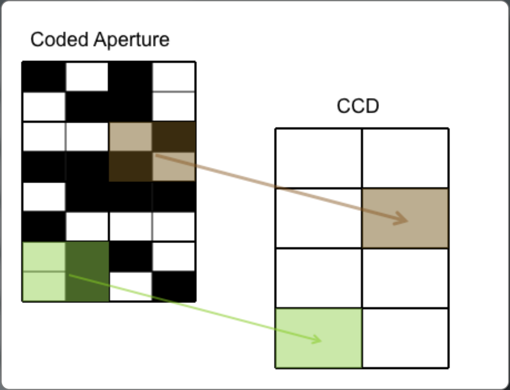


- A row of Φ consists of the vectorized $N \times N$ randomly generated binary array determined by the digital micromirror device (DMD).
- A single "snapshot" consists of an $N \times N$ image multiplied by a row of Φ.
- M "snapshots" means there are M rows/samples recorded.
- $M \gg 16$ for high resolution images.

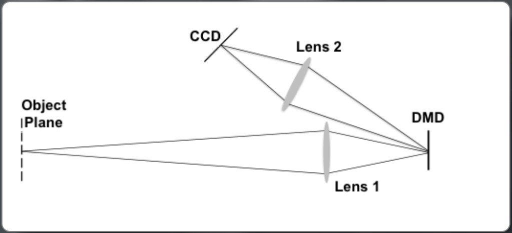
Camera Design Concept

We desire to capture high resolution images in 16 or fewer "snap-shots" to decrease aquisition time. This can be done by distributing the work to many photon detectors. In particular, we can leverage low cost charge-couple devices (CCD's) to create a cost-effective high resolution camera.

- High resolution DMD maps 4×4 or greater pixel elements into one CCD element.
- Coded aperture patterns should avoid delta function elements due to energy sensitivity issues.



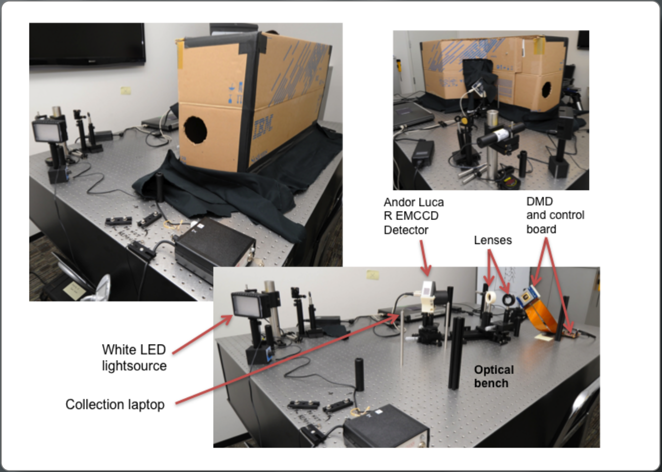
Experimental Setup



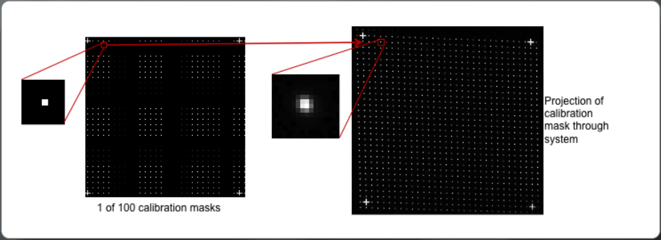
- Thermoelectrically cooled CCD operating at $-20\,^{\circ}$C.
- Two achromatic doublet imaging lenses.
- HD format digital micromirror device (DMD) with computer interface.

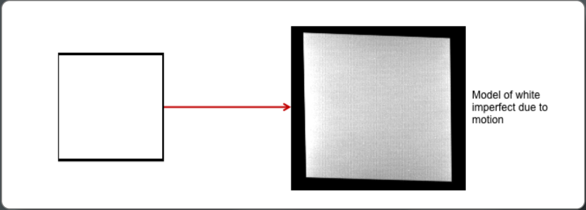
Camera Design Concept

Experimental Setup



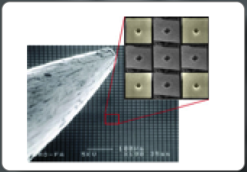
Calibration Issues

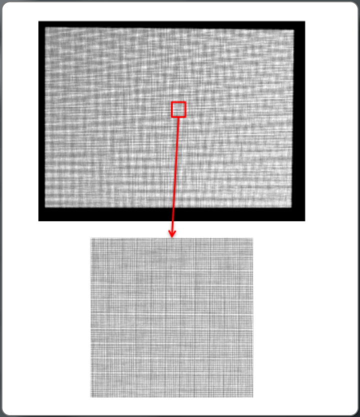




Camera Design Concept

Unexpected Issues





- Grid pattern may be due to tiny repetative motion captured during data collection.
- Other sources of error include modeling of Φ and noise.

General Estimation Techniques

- L_1 Minimization
- Matching Pursuit
- Iterative Thresholding
- Total-Variation Minimization

$$\arg\min_{\alpha_0'} \mathrm{TV}(\alpha_0') \approx \|\nabla \alpha_0'\|_1 \text{ subject to } y = \Theta\alpha_0'$$

Effective Methods for Camera Design

An Iterative thresholding routine based on image separations (to be explained next).

An estimate found by using both TV and Besov regularizers by solving

$$\arg\min_{\alpha_0'} \| \nabla \alpha_0' \|_1 + \| W\alpha_0' \|_1 \quad \text{subject to} \quad \|y - \Theta\alpha_0'\|_2 < \epsilon,$$

where W is an orthogonal wavelet transform (Haar). This is done by using the Split Bregman Algorithm.

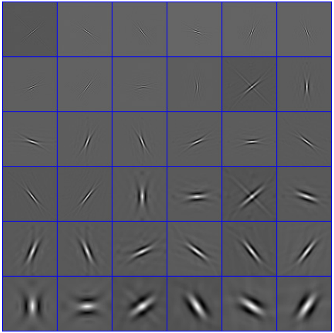
Reconstruction Algorithms

Given $M_p, M_t \geq N^2$, the dictionary $D_p \in \mathbb{R}^{N^2 \times M_p}$ and $D_t \in \mathbb{R}^{N^2 \times M_t}$ are chosen such that they provide sparse representations of piecewise smooth and texture contents, respectively.

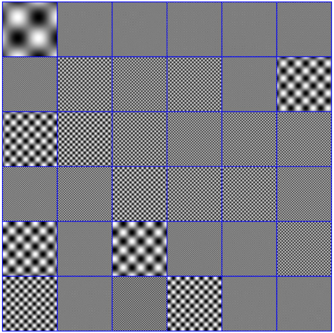
Examples

- D_p can be a wavelet or a shearlet frame dictionary.
- D_t can be a DCT or a Gabor dictionary.

Reconstruction Algorithms



Atoms from a shearlet dictionary.



Atoms from the DCT dictionary.

Reconstruction Algorithms

We propose to recover the image x by estimating the components x_p and x_t as $D_p\hat{\alpha}_p$ and $D_t\hat{\alpha}_t$ given that

$$\hat{\alpha}_p, \hat{\alpha}_t = \arg\min_{\alpha_p,\alpha_t} \lambda\|\alpha_p\|_1 + \lambda\|\alpha_t\|_1 + \frac{1}{2}\|y - A_p\alpha_p - A_t\alpha_t\|_2^2,$$

where $A_p = \Phi D_p$ and $A_t = \Phi D_t$. By setting $A = [A_p, A_t]$, we can divise an iterative reconstruction method as follows.

Reconstruction Algorithms

The objective function can then be re-written as

$$w(\alpha) = \lambda\|\alpha\|_1 + \frac{1}{2}\|y - A\alpha\|_2^2 \qquad (1)$$

where α contains both the piecewise smooth and texture parts. Let

$$d(\alpha, \alpha_0) = \frac{c}{2}\|\alpha - \alpha_0\|_2^2 - \frac{1}{2}\|A\alpha - A\alpha_0\|_2^2, \qquad (2)$$

where α_0 is an arbitrary vector of length N^2 and the parameter c is chosen such that d is strictly convex.

This constraint is satisfied by choosing

$$c > \|A^T A\|_2 = \lambda_{\max}(A^T A),$$

where $\lambda_{\max}(A^T A)$ is the maximal eigenvalue of the matrix $A^T A$. Adding (2) to (1) gives the following surrogate function

$$\tilde{w}(\alpha) = \lambda\|\alpha\|_1 + \frac{1}{2}\|y - A\alpha\|_2^2 + \frac{c}{2}\|\alpha - \alpha_0\|_2^2 - \frac{1}{2}\|A\alpha - A\alpha_0\|_2^2.$$

This surrogate function $\tilde{w}(\alpha)$ can be re-expressed as

$$\tilde{w}(\alpha) = a_0 + \frac{\lambda}{c}\|\alpha\|_1 + \frac{1}{2}\|\alpha - x_0\|_2^2, \tag{3}$$

where

$$x_0 = \frac{1}{c}A^T(y - A\alpha_0) + \alpha_0$$

and a_0 is some constant.

Reconstruction Algorithms

Let a_+ denote the function $\max(a, 0)$. Given that

$$\mathcal{S}_\lambda(x) = \frac{x}{|x|}(|x| - \lambda)_+$$

is the element-wise soft-thresholding operator with threshold λ, the global minimizer of the surrogate function (3) is given by

$$\begin{aligned}\alpha_{sol} &= \mathcal{S}_{\lambda/c}\left(x_0\right) \\ &= \mathcal{S}_{\lambda/c}\left(\frac{1}{c}A^T(y - A\alpha_0) + \alpha_0\right).\end{aligned}$$

Reconstruction Algorithms

It can then be shown that the iterations

$$\alpha^{k+1} = \mathcal{S}_{\lambda/c}\left(\frac{1}{c}A^T(y - A\alpha^k) + \alpha^k\right)$$

converge to the minimizer of the function w in (1).

By breaking the above iteration into the two representation parts, we get:

Reconstruction Algorithms

Reconstruction Algorithm

Initialization: Initialize $k = 1$ and set $\alpha_p^0 = 0$, $\alpha_t^0 = 0$ and $r^0 = y - A_p\alpha_p^0 - A_t\alpha_t^0$.

Repeat:

1. Update the estimate of α_p and α_t as

$$\alpha_p^k = \mathcal{S}_{\lambda/c}\left(\frac{1}{c}A_p^T(r^{k-1}) + \alpha_p^{k-1}\right)$$
$$\alpha_t^k = \mathcal{S}_{\lambda/c}\left(\frac{1}{c}A_t^T(r^{k-1}) + \alpha_t^{k-1}\right).$$

2. Update the residual as

$$r^k = y - A_p\alpha_p^k - A_t\alpha_t^k.$$

Until: stopping criterion is satisfied.

Reconstruction Algorithms

Lin. Bregman Algorithm

Initialization: Initialize $k = 1$ and set $\alpha_0^0 = 0$, $\beta^0 = 0$, and $r^0 = y - A\alpha_0^0$.

Repeat:

1. Update the estimate of α_0 by the following iterations

$$\begin{aligned} \beta^k &= \beta^{k-1} + A^T(r^{k-1}), \\ \alpha_0^k &= \lambda \mathcal{S}_\mu \left(\beta^k \right). \end{aligned}$$

2. Update the residual as

$$r^k = y - A\alpha_0^k.$$

Until: stopping criterion is satisfied.

Reconstruction Algorithms

Gen. Split Bregman

While $\|\alpha_0^k - \alpha_0^{k-1}\| > tol$,

for $n = 1$ to N

$\alpha_0^{k+1} = \min_u H(u) + \frac{\lambda}{2}\|d^k - F(u) - b^k\|_2^2$

$d^{k+1} = \min_d \|d\|_1 + \frac{\lambda}{2}\|d - F(u^{k+1}) - b^k\|_2^2$

end

$b^{k+1} = b^k + (F(u^{k+1}) - d^{k+1})$

end

Experimental Results

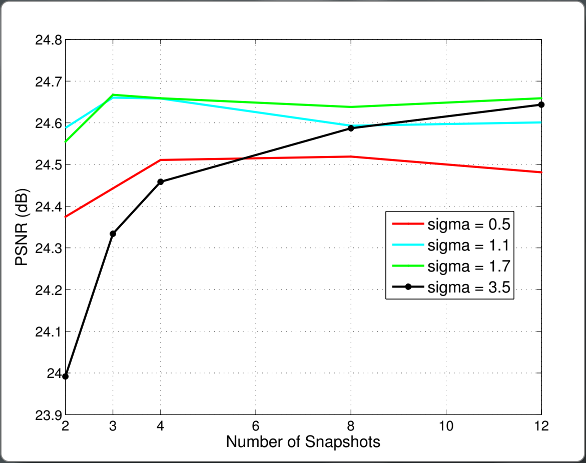
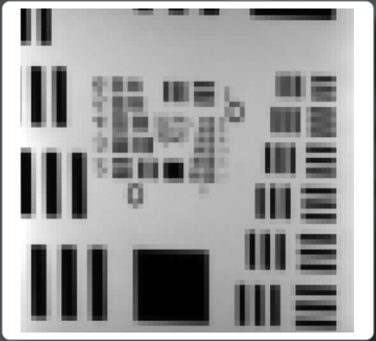
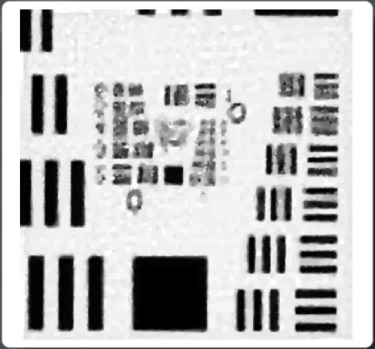


Figure: *The PSNR as a function of snapshots for experiments with the Boats image for differing amounts of noise.*

2 Snapshots

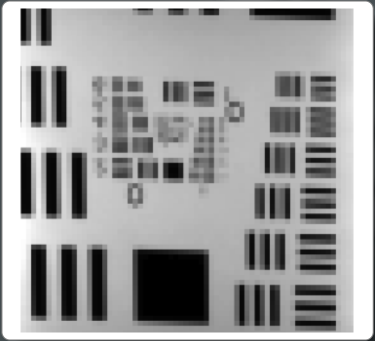


Raw CCD Capture

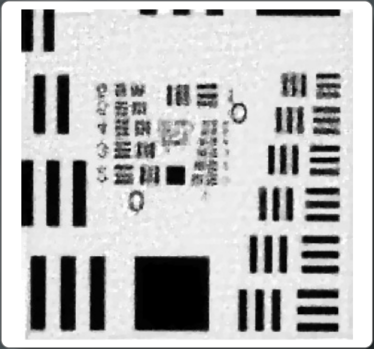


CS Reconstruction

4 Snapshots

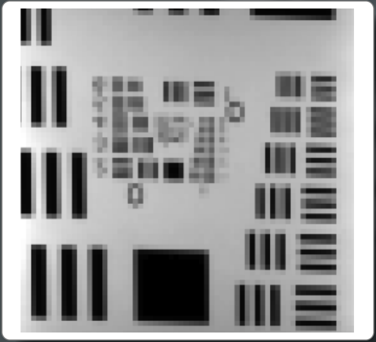


Raw CCD Capture

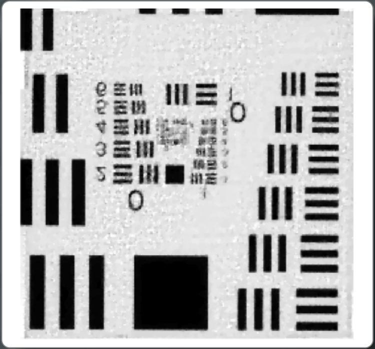


CS Reconstruction

8 Snapshots



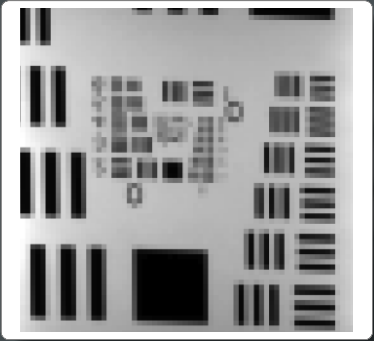
Raw CCD Capture



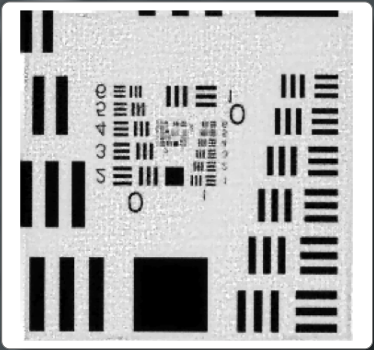
CS Reconstruction

Experimental Results

16 Snapshots



Raw CCD Capture



CS Reconstruction

2 Snapshots

Raw CCD Capture

CS Reconstruction

4 Snapshots

Raw CCD Capture

CS Reconstruction

Experimental Results

8 Snapshots

Raw CCD Capture

CS Reconstruction

16 Snapshots

Raw CCD Capture

CS Reconstruction

References

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