Constrained Optimization Applied To Pulse Compression Codes, And Filters

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1. INTRODUCTION

When choosing a pulse compression code or codes for a given task many issues must be considered. For example if the application is a radar designed for a scenario dominated by distributed clutter the total power from the side lobes, also known as the integrated side lobe level (ISL), is very important. On the other hand if the application requires detection of targets in the presence of large clutter discretes then the peak side lobe level (PSL) is more important. If the desired ISL or PSL performance cannot be achieved with a matched filter some SNR gain may be sacrificed and a mis-matched filter may be used to achieve the desired side lobe levels.

A number of different code options are currently available. These include continuous phase codes, poly-phase codes, and bi-phase codes. Classical continuous codes are linear (LFM) [1] and non linear (NLFM) [2] frequency modulation codes. Poly-phase options include such codes as Lewis and Kretschmer's P4 code [3], Gartz's uniform amplitude codes [4], or codes that can be found using the constrained optimization approach of Nunn and Welch [5]. Widely used bi-phase codes include Barker codes [6], and Pseudo Random Noise (PRN) codes [7]. Mismatched filters have also been widely discussed. Ackroyd and Ghani [8] describe ISL optimized filters, and Baden and Cohen [9] describe PSL optimized filters.

The purpose of this paper is to give the reader an understanding of the expected results when using constrained optimization [10] to find codes that are optimized for auto correlation ISL, auto correlation PSL, mismatch filter ISL and mismatch filter PSL. Concentrating mainly on 32 chip codes this paper tries to give a feel for the depth and breadth of possibilities for finding different pulse compression codes and filters.

Section (2) defines a framework within which each optimization is performed. Section (3) shows various results including example codes and numbers of expected codes for various sizes indicating abundance.

2. AN OPTIMIZATION FRAMEWORK

Each optimization in this paper is a special case of the following standard constrained optimization problem

$$\text{Minimize } f(x)$$

$$\text{subject to} \quad g_i(x) \le k_i \text{ for } i=1..L. \tag{1}$$

Problems which are couched in this framework can be solved using the methods found in many references, for example [10].

In this paper a typical cost or objective function f(x) would be a function that describes, for example, the ISL of the filtered code, and the individual constraints function $g_i(x) \le k_i$ could among other possibilities consist of constraints that the individual side lobes not exceed a certain value along with another constraint that the filter loss remain bound by some other value.

Here is a specific example that illustrates how a code/filter pair with minimum ISL and with filter losses less than a constant can be found: Let c_i for i= 1..n be a set of complex numbers. If this set of complex numbers is of constant amplitude then they can correspond to the individual chips of an n length pulse compression code which is denoted as c. Another set of complex numbers f_i for i=1..m can correspond to the filter coefficients of length m which is denoted as f.

Let $R_i(c,f)$ for i=1..(n+m-1) be the cross correlation of the normalized code c (the normalization is explicitly part of the function) with the filter f, having its correlation peak in the first position then

$$ISL(c,f) = \sum_{i \ne 1} \left(\frac{R_i}{R_1} \right)^2 \tag{2}$$

and the filter loss would be

$$Loss(c,f) = \frac{\sum_{ii=1..n} \left(c_i c_i^* \right) \sum_{ii=1..m} \left(f_i f_i^* \right)}{R_1^2} \tag{3}$$

The problem of finding a code/filter pair with minimum ISL, with loss less than or equal to L and a maximum correlation peak of 1 becomes

$$\text{minimize} \quad ISL(c,f)$$

$$\text{subject to} \quad Loss(c,f) \le L \text{ , } R_1(c,f)=1 \tag{4}$$

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The resulting set {c,f} is a code/filter pair. A code/filter pair that is found using this strategy can be called a locally optimal code/filter pair, or if searching for matched filter codes a locally optimal code.

3. RESULTS

In this section results from six different sorts of optimizations are shown. These are as follows: Matched filter optimizations for both ISL and PSL performance, mismatched filter optimizations where the code is fixed and the filter varies for ISL and PSL performance, and mismatched filter optimizations where both the code and the filter vary for ISL and PSL performance.

a. Matched Filter Optimization for ISL Performance. This was done first for two reasons. At about 400 32 element optimizations per second on a 2.8 GHz Pentium computer it is the fastest applicable optimization of the six aforementioned types to perform, and the best ISL optimized codes can be used as starting points for other optimizations.

The first step was to pick a variety of random starting points and find the nearest local optimum. Locally optimal codes with unique side-lobe structures were retained. As expected the number of unique locally optimal code outputs grew at an exponential rate with respect to code size. The code size versus the number of codes is shown in figure (1).

We decided to concentrate mainly on 32 element codes, so a search was made to find ISL optimized matched filter codes of length 32. More than a million unique codes of this type were found and a plot of the ISL versus PSL performance of these codes is shown in figure (2).

b. Matched Filter Optimization for PSL performance. The codes shown in figure (2) were used as starting codes for various other optimizations. The first type of optimization was to find good PSL matched filter codes. A search was made for Barker level codes, i.e. codes for which the peak side lobe to main lobe ratio is equal to 1/n where n is the size of the code. Ninety eight such 32 element codes were found (four 64 element Barker level codes were also found). Plots of an original ISL optimized code, and the PSL optimized code that was found using the ISL optimized code as a starting point, are shown for the best of these 32 element Barker level codes in figure (3). Table (1) contains the phase codes that correspond to the plots in figure (3). The ISL and PSL of the ISL optimized code in figure (3) are -15.60 dB and -28.42 dB, while the ISL and PSL of the PSL optimized code are -15.43 dB and -30.10 dB. Table 2) contains the phases for four 64 element Barker level codes with ISL values of -16.17, -16.12, -16.05, and -16.08 respectively.

c. Mismatched Filters for ISL and PSL Performance. A search was then conducted to find mismatched filters for ISL then PSL performance for the ISL optimized matched filter codes described above. Two of the best examples of these code/filter pairs are shown in figures (4) and (5). The ISL, PSL, and filter loss of the code/filter pair in figure (4) are -24.69 dB, -34.79 dB and 0.19 dB. The ISL, PSL and filter loss for the code in figure (5) are -21.17 dB, -40.00 dB, and 0.21 dB.

d. ISL and PSL Optimization by Varying both code and filter at the same time. In this step ISL and PSL performance, for pairs in which the code and filter were both allowed to evolve in the optimization process, were found. The loss was constrained to be no more than 1 dB. Excellent examples of codes found with this method are shown in figures (6) and (7). For the code and filter pair in figure (6) the ISL, PSL and loss were -36.29 dB, -47.47 dB, and 0.99 dB. The values for the code and filter pair in figure (7) were -32.19 dB, -50.54 dB, and 0.97 dB.

Figure (8) shows the progression of the 25000 best ISL pairs. The portion on the far right shows results when optimizing for matched filter performance. The middle portion depicts the performance for mismatched filters when only the filter is optimized for, and the code is chosen from the available locally optimal matched filter codes. On the left are the results of code/filter pairs where the code and filter are optimized together with the constraint that the filter loss be less than 1 dB.

4. SUMMARY

Using the constrained optimization approach this paper exhibits the potential for finding excellent pulse compression codes, and code and filter pairs. For codes of size 32 element or larger there seems to exist a multitude of code and filter pairs with side-lobe levels consistent with the needs of modern radars.

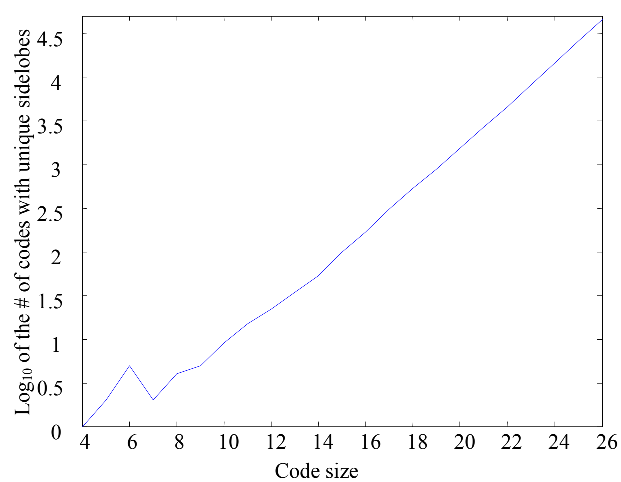

Figure 1 A comparison of code size versus the log of the number of unique codes for that size.

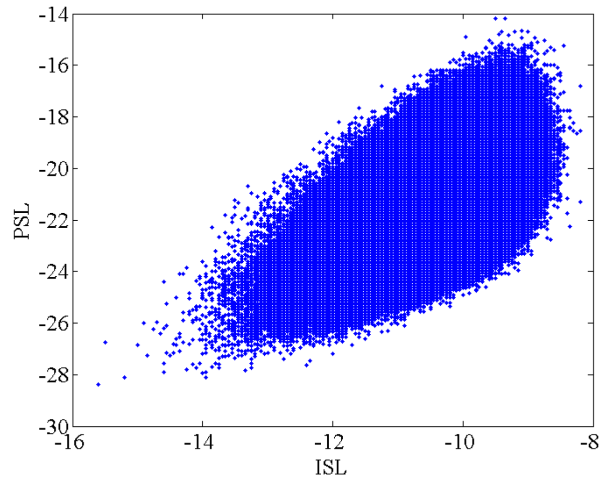

Figure 2 A plot of ISL versus PSL performance for more than one million 32 element ISL optimized matched filter codes with unique side-lobe structures.

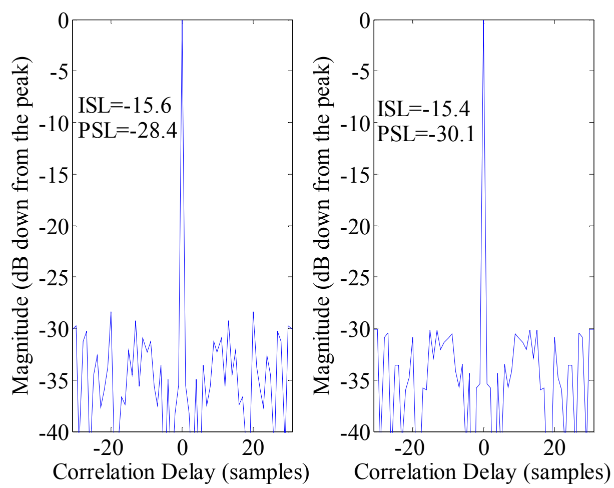

Figure 3 On the left is a plot of the side-lobe structure of the best ISL optimized matched filter, 32 element code. On the right is the Barker level code that is found with further processing.

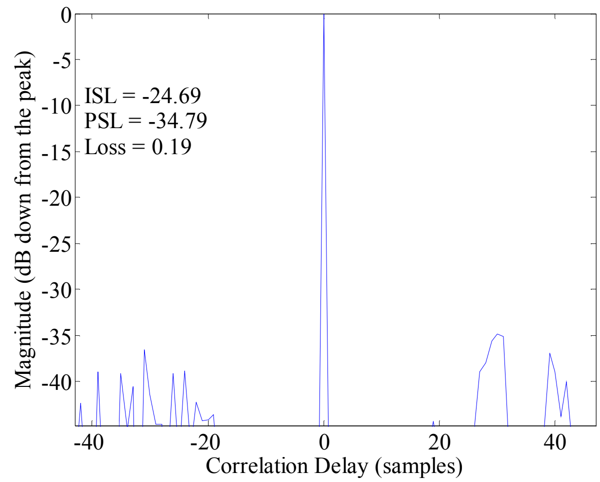

Figure 4 The best ISL optimized code/filter pair generated by keeping the code fixed.

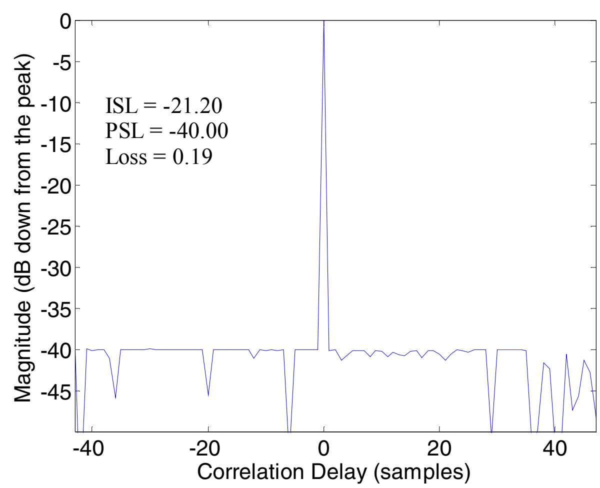

Figure 5 An excellent code/filter pair, optimized for PSL while keeping the code fixed.

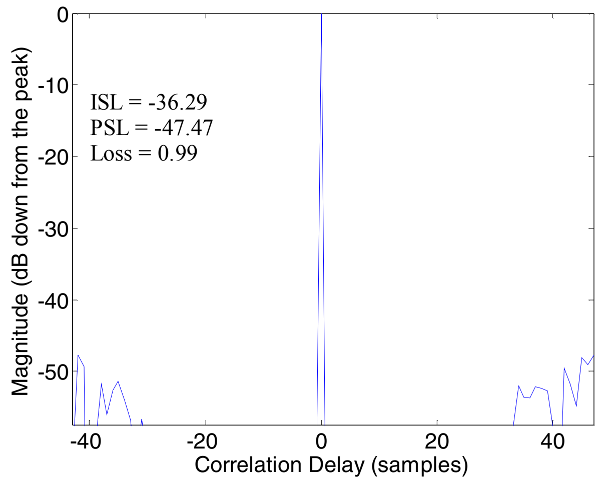

Figure 6 An excellent code/filter pair, optimized for ISL with both code and filter varying.

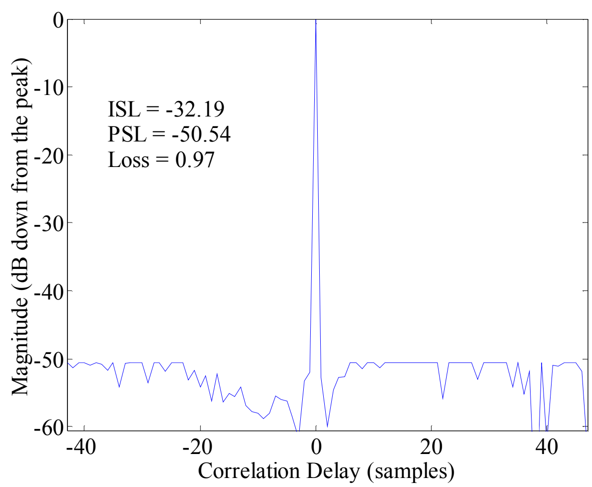

Figure 7 An excellent code/filter pair, optimized for PSL with both code and filter varying.

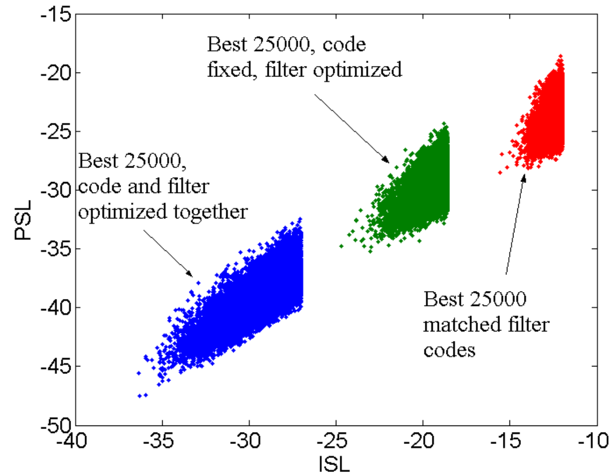

Figure 8 ISL versus PSL performance for matched filter codes, filter optimized code and filter pairs, and code/filter optimized code and filter pairs where the loss is constrained to be less than 1 dB.

Table 1 A table containing the best ISL optimized matched filter code of length 32, and the Barker level code that was found by further processing.

Code 1	Code 2
0	0
0.0246	0.0015
0.1716	0.1151
2.0924	2.0809
2.1146	2.0483
1.9021	1.9035
0.3746	0.3360
0.9975	0.9962
0.7064	0.7008
2.7216	2.7086
-2.3282	-2.3754
2.8747	2.9093
0.2955	0.2685
0.5391	0.5285
-1.2042	-1.2031
1.6900	1.6784
0.1615	0.1419
0.4964	0.4929
-1.7856	-1.8087
-2.9569	-2.9830
-0.7736	-0.7856
-2.0034	-2.0589
1.9271	1.9203
-1.3251	-1.3688
2.7824	2.8277
0.8172	0.7923
1.7742	1.7939
-2.5050	-2.5332
-0.1188	-0.1426
1.8322	1.8208
-2.0683	-2.0959
0	0

Table 2 Phases for four unique 64 element Barker level codes.

Code 1	Code 2	Code 3	Code 4
0	0	0	0
-0.0115	0.1361	0.2215	0.0841
0.1902	0.4896	0.3963	0.9151
0.2444	1.2925	0.7529	0.5378
-0.1855	2.5217	0.8040	-0.5086
-1.3068	3.0875	-0.1715	-1.2577
-1.8570	2.8114	-0.4847	-0.1352
-2.4399	2.1123	-0.6939	-0.4703
-1.8904	1.9987	-1.1226	-1.2559
-2.5578	-2.6543	-0.9295	-0.9781
-1.9831	-1.8271	-0.4303	-0.9836
0.9501	-1.0399	0.9335	2.6954
0.5265	-1.0640	1.4655	2.1823
0.8568	-2.4744	2.0923	-0.9253
2.6382	2.9827	2.4428	-1.5544
2.5707	2.0777	-3.1128	1.9222
-2.4412	2.4530	1.5958	0.9086
-2.0125	-2.3577	0.3472	-0.3227
-2.4328	-1.5527	-2.4339	-0.1831
1.6052	-0.0674	-1.0890	0.3575
2.6297	0.0458	-1.9348	2.6109
-1.5630	-3.0091	1.7611	2.7577
-2.6530	-3.1305	-1.5022	-0.9293
-1.7973	-0.5764	2.9696	-0.4233
-1.0147	-1.5165	-0.0444	1.7518
-0.8345	-2.9447	-1.6020	1.9530
2.7373	-2.2776	0.7348	1.8941
-2.2317	-1.8392	1.4939	2.6948
0.7390	2.4165	2.2210	-3.0171
-1.0448	2.9090	-1.1492	-1.7188
0.1137	2.1048	-0.6600	-0.0748
-2.9445	1.7633	-2.6192	-2.5615
-0.9023	-1.5974	2.5358	-2.5265
1.7292	-2.9607	1.2342	-0.9025
-2.7714	1.6708	1.8171	-0.4885
-0.0983	0.9138	-1.0377	-3.0567
1.6125	1.8468	-2.2135	0.7154
-2.3506	-0.5822	0.5994	1.2435
-0.8582	-1.3905	-0.5068	0.3585
-0.8580	2.4131	-2.0387	-2.4147
2.3461	1.7135	2.5211	-0.6333
3.1157	-0.7852	1.4648	-2.9787
-0.0556	-0.4770	-1.8187	-2.0400
-0.1864	2.4836	0.3267	2.3885
-0.5984	-1.5204	0.4625	0.8858
-2.6859	0.9242	-1.9016	0.6393
2.7312	2.7196	-3.0048	-1.2461
-0.2450	-1.5760	2.1387	1.9749
-0.0148	0.3626	-0.7122	-1.3956
2.2964	-2.7099	1.6564	2.9234
-0.7800	1.2621	-2.4365	0.7847
-1.9556	-1.1489	-0.3816	-2.7813
1.8933	3.0569	1.1016	1.0345
0.7016	0.6002	-2.8793	-1.4713
-1.2587	-2.1178	-1.2771	2.6927
2.9475	0.9215	2.2655	-0.6129
0.3301	-2.2584	-0.1841	1.3032
-2.5362	1.4601	2.2916	-2.3728
0.7459	-1.4943	-1.4257	-0.2037
-2.0389	1.2213	1.4270	2.6681
1.1941	-2.6482	-1.2957	-1.9145
-1.3078	0.2006	1.6799	0.4690
2.5857	3.0859	-2.4342	3.0732
0	0	0	0

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