Polyphase Pulse Compression Codes with Optimal Peak and Integrated Sidelobes

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***Abstract*—Near-optimal or globally optimal integrated sidelobe level (ISL) polyphase codes are found for lengths 46 through 80 by using a stochastic optimization technique. Polyphase Barker codes are found for lengths 64 to 70, and 72, 76 and 77 using constrained optimization starting at optimal-ISL codes.** [1]

Keywords: Barker sequence, peak sidelobe level, polyphase Barker, stochastic optimization.

I. Introduction

Pulse compression coding is used in radar applications to gain the signal-to-noise (SNR) benefits of a long pulse along with the range resolution of a short pulse. An important figure of merit used to describe pulse compression codes is the peak sidelobe level (PSL). Codes with good PSL can be used to discriminate returns of interest from unwanted close-in discrete returns.

Perhaps the most well-known phase codes used in radar pulse compression are the binary Barker codes. These are codes with elements either 1 or -1 for which the PSL-to-peak voltage ratio is $1/N$, where N is the length of the code. Barker codes exist for several lengths up to 13, but no Barker codes exist for odd lengths over 13 [1]. It is generally accepted that there are no such biphase codes of even length greater than 4. Furthermore, exhaustive searches have been conducted to establish that the longest codes with PSL-to-peak ratios of $2/N$ and $3/N$ are probably length 28 and 53 respectively (refer, e.g., to [2] and [3]). Codes achieving $4/N$ sidelobe levels have been found at lengths up to and including 82 [4].

In 1965 Golomb and Scholtz [5] started to look at more general sequences called generalized Barker sequences which obey the same PSL-to-peak ratio maximum, and have complex elements not necessarily of uniform size. Here we will focus on codes that have elements of uniform size. We refer to them as ”complex uniform”, or polyphase codes. It has become a challenge to extend the list of lengths for which such sequences are known. Zhang and Golomb found polyphase Barkers up to length 19 in 1989 [6]. Friese and Zottman [7] extended the list to length 31 in 1994. Brenner [8] found codes up to length 45 in 1998. Borwein and Ferguson [9] found codes of length up to 63 in 2005. Also in 2005, Nunn [10] found four polyphase Barker codes of length 64. Borwein and Ferguson [11] subsequently presented a polyphase Barker of length 65 in 2007.

Whereas one goal of this paper is to find yet longer polyphase Barker sequences, much of the effort is concentrated on finding codes with very good integrated sidelobe levels

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(ISL), defined as

$$\mathrm{ISL} = 10 * \log_{10} \sum_{\mathrm{i} \neq 0} \left(\frac{\mathrm{R_i}}{\mathrm{R_0}} \right)^2 ,$$

where the R_i are the elements of the autocorrelation sequence, $i = 1 - N, \ldots, N - 1$, and R_0 is the autocorrelation peak [12]. This is done because long codes cannot have Barker-level sidelobes without also having good ISL. With this in mind good ISL codes are found, and then used as starting points for local searches for low-PSL codes.

In a previous publication [10], one of the authors of this paper indicated methods for finding local minima for objective functions related to pulse compression figures of merits such as matched filter ISL and PSL. These methods were used in conjunction with a stochastic search to find large numbers of 32-element and 64-element codes with locally optimal ISL. These codes were then used as initial conditions for length-32 and length-64 Barker searches. The methodology used in this paper is similar to the method of Borwein and Ferguson [9]. They used a different local optimization strategy with a stochastic search to find polyphase Barker codes for lengths from 46 to 63. They also used a variation of the Inverse Coupon Collectors problem to establish, with 99 percent certainty, the equivalent of minimum possible ISL values (refer, e.g., to [13] and [14]).

It is one purpose of this paper to use the local optimization/stochastic search strategy to extend the list of known lengths for which polyphase Barker codes may be found. In addition this paper will attempt to give a feel for the overall number of available Barkers.

II. Discussion of Methods and Results

To find polyphase Barker codes, the authors of this paper used an existing set of routines which were designed to find large numbers of good ISL codes. These routines evolved over a number of years, and have been used effectively to find many codes of various lengths both binary and polyphase. It would be difficult to completely describe the methodology of these tools, but the basic methodology is as described in Borwein and Ferguson's paper i.e. the use of a local optimization strategy combined with a stochastic, or global optimization strategy.

For each number of chips from 10 through 80 the program was run for a substantial amount of time on a a set of computers containing one or more Intel or AMD processors. These searches were performed off and on, in the background for many months. Computers that have been used in these searches include a computer with a 2.4 GHZ Intel quad-core processor, a 3.4 GHz Intel Pentium D processor, a pair of 2.8 GHz single core processors, and a Beowulf cluster combining 18 single core 2.2 GHz Opteron processors.

The work in this paper consisted of three tasks or objectives. The first part was to establish the length of the longest polyphase Barker code attainable with the routines and processing power available to the project. The second objective which was consistent with, and accomplished concurrently with, the first was to apply sophisticated searches and significant computational effort to seek low, and possibly optimally low, ISL values for codes from length 46 through 80. The third part was to partially understand the quantity and nature of available polyphase Barkers.

To accomplish the first and second task, existing routines were run for a substantial amount of time for each length from 10 through 80 to find large numbers of excellent ISL codes. The amount of effort used at each of these lengths was consistent with, and in most cases exceeded, the level of effort Borwein and Ferguson used to establish the equivalent of best ISL values for lengths up to 45 with 99 percent certainty. The codes which eventually yielded the polyphase Barker codes of lengths 65 through 70 in this paper were found in a relatively short time on a pair of desktop computers. The effort was continued and extended in order to search for better ISL codes and further polyphase Barker codes, and in the case of the codes from length 10 through 52, to ensure that as many polyphase Barkers as possible were found. This continued search ultimately yielded a polyphase Barker code at each of the lengths 72, 76 and 77.

The best ISL values found for lengths 41 through 80 along with the PSL values of the codes with the best ISL are shown in Table 1. The results for lengths less than 64 are compared to the results reported in [9]. These results were always at least as good as, and often better than, the comparison results. For lengths 64 and above, there are no comparison results. Though some of the improvements exhibited in Table 1 appear small, it should be noted that the point of this table is to find the lowest ISL values ever found.

The codes resulting from this search were used as starting points for a constrained optimization search, as described in [10], with the objective of finding polyphase Barker codes. This attempt was successful for all lengths below 71, and for lengths 72, 76, and 77. Table 2 gives the ISL values found for sequences used as starting points for searches that resulted in Barker sequences of length 70, 72, 76 and 77.

Table 1. ISL and PSL of Lowest-ISL Sequences Found

N	ISL (dB)	PSL (dB)	Min ISL (dB) in [9]
41	-17.19	-29.84	-17.19
42	-16.14	-29.50	-16.14
43	-16.63	-27.38	-16.57
44	-16.44	-31.10	-16.44
45	-16.53	-29.67	-16.53
46	-17.11	-29.72	-16.30
47	-17.98	-29.94	-17.98
48	-16.70	-29.45	-16.70
49	-17.25	-26.48	-16.97
50	-16.78	-30.77	-16.57
51	-18.57	-31.25	-17.61
52	-16.97	-30.93	-16.48
53	-17.47	-32.53	-17.05
54	-17.63	-31.43	-16.50
55	-18.22	-30.59	-17.22
56	-17.08	-30.74	-17.00
57	-18.02	-29.49	-16.63
58	-17.28	-29.85	-17.28
59	-17.99	-29.92	-16.69
60	-17.32	-31.37	-16.88
61	-17.28	-32.54	-16.64
62	-17.46	-30.99	-16.83
63	-17.62	-32.39	-16.87
64	-17.45	-31.13	-17.36
65	-18.37	-32.22	◇
66	-17.70	-30.48	◇
67	-18.06	-28.91	◇
68	-18.08	-32.41	◇
69	-17.61	-31.67	◇
70	-18.08	-31.78	◇
71	-17.78	-29.44	◇
72	-17.66	-33.83	◇
73	-18.00	-30.60	◇
74	-17.53	-33.25	◇
75	-17.90	-30.74	◇
76	-17.93	-33.58	◇
77	-18.35	-31.46	◇
78	-17.77	-32.54	◇
79	-17.80	-32.06	◇
80	-17.69	-34.44	◇

Four Barker sequences were found for length 70. Barkers were also found for lengths 65 through 69. The resulting polyphase Barker codes are shown in Tables 3a, 3b, and 3c. No polyphase Barker codes were found for lengths 71, 73, 74, and 77 through 80.

Table 2. Starting Point ISL and PSL		
N	ISL (dB)	PSL (dB)
70	-18.08	-31.78
70	-17.68	-31.91
70	-17.41	-31.44
70	-17.35	-32.06
72	-17.27	-33.24
76	-17.93	-33.58
77	-17.84	-33.48

Each of the nine codes in Tables 3a, 3b, and 3c have zero phase in the the first two elements. This is a standard form for polyphase codes which can be achieved for a given code by applying some combination of four PSL-preserving transformations [15]. In order to save space, these zero phase elements were not displayed.

Table 3a. New Polyphase Barkers			
	Length		
i	65	66	67
3	0.669	0.651	0.250
4	0.705	1.094	-0.361
5	1.620	1.098	-1.531
6	2.788	0.760	-1.742
7	-2.279	1.194	-1.431
8	-0.927	0.695	-1.171
9	-0.873	-0.238	-1.722
10	-1.549	0.359	3.130
11	-0.016	1.129	2.805
12	0.796	2.068	-2.514
13	-3.044	2.624	-1.989
14	-1.513	-2.402	-0.967
15	-0.131	-0.501	-0.748
16	1.590	-0.828	0.023
17	-3.074	-1.854	0.234
18	2.644	-0.840	-1.041
19	2.095	0.230	-2.578
20	-3.042	2.393	1.586
21	2.130	-3.060	0.326
22	1.384	1.131	1.287
23	-2.918	-0.166	0.641
24	-2.928	-1.944	-1.280
25	3.034	2.866	2.454
26	1.542	-2.745	1.149
27	0.033	-2.547	-1.694
28	-2.990	0.957	-2.947
29	-2.381	1.582	1.998
30	-1.370	-0.442	0.742
31	2.276	0.103	-1.043
32	1.634	1.712	-1.372
33	-0.725	-0.986	0.169
34	-1.637	-1.787	2.094
35	-1.782	1.166	2.940
36	2.485	-1.647	-2.672
37	1.889	-1.843	0.929
38	-2.605	2.849	-0.487
39	1.191	2.671	-2.745
40	1.564	1.365	-2.633
41	1.478	-0.730	-0.853
42	0.231	0.377	0.507

Table 3a, continued			
	Length		
i	65	66	67
43	-0.868	2.694	1.177
44	-2.381	2.546	-1.797
45	1.418	-0.056	1.766
46	-2.004	-0.933	-1.942
47	-2.523	3.130	2.306
48	2.127	0.163	-0.263
49	1.523	2.659	-3.031
50	-1.510	-2.094	-0.351
51	1.840	0.546	-3.107
52	-1.345	-1.692	-1.176
53	2.376	0.850	1.226
54	0.815	2.688	1.354
55	-2.078	-2.075	-2.455
56	0.387	1.289	-0.458
57	2.949	-1.056	2.348
58	-0.379	2.784	-2.290
59	2.532	0.931	0.382
60	-1.659	-2.703	-2.872
61	1.053	0.141	0.618
62	-2.749	-2.331	-2.105
63	0.520	0.424	0.139
64	-2.746	2.853	1.967
65	0.824	0.024	-2.217
66		-2.306	0.454
67			-3.091

Table 3b. New Polyphase Barkers			
	Length		
i	68	69	70
3	0.4067	0.07976	1.42523
4	-0.5046	0.58296	2.08816
5	-2.1282	1.81485	-3.05258
6	-2.0912	1.82098	-2.77191
7	-1.7483	1.00855	-2.45249
8	2.8677	0.89255	-2.46227
9	1.7844	0.29756	-2.24951
10	2.0184	-0.1533	-0.37481
11	1.5365	0.64288	0.13029
12	1.0150	0.69489	2.02962
13	1.0508	1.43504	2.1759
14	2.2799	2.14618	-1.16998
15	1.9425	-2.20297	-0.38929
16	2.6898	-1.69645	0.65086
17	1.7585	-0.50078	1.61698
18	1.6239	-1.0920	1.80808
19	1.9010	-1.69751	0.10839
20	2.9506	-1.8551	0.3322

Table 3b, continued			
	Length		
i	68	69	70
21	-2.5654	-2.67005	0.30113
22	-2.0949	2.01319	0.06233
23	-0.7560	1.26739	-1.16039
24	0.6002	1.43274	-2.26534
25	0.4148	0.47052	-2.67878
26	1.9365	-0.75635	-2.41804
27	0.4734	2.38315	2.01812
28	-2.6279	2.35055	0.66465
29	2.1562	-1.98574	-2.15192
30	-0.0834	-0.85718	3.04328
31	-2.9324	-0.54268	0.13284
32	1.1235	2.63622	-1.15376
33	-0.6224	2.65743	-2.66255
34	2.7057	-1.48012	3.08902
35	-0.4964	1.16378	0.38185
36	2.1708	1.01598	-1.61902
37	-1.1933	-1.5716	-2.00515
38	2.1368	0.83961	-2.09872
39	0.3980	2.20069	-2.48749
40	-2.3617	-2.18334	2.2047
41	-0.9185	-1.0478	-2.32013
42	2.4076	2.60719	0.27012
43	-0.5770	1.49634	-1.40224
44	1.1069	0.19691	-2.17547
45	2.6485	-2.1478	2.10378
46	-0.5231	-2.44501	-1.30729
47	2.6271	1.10051	1.15889
48	-1.4287	0.36299	-1.32073
49	0.7318	-1.58034	-1.99762
50	-2.2603	2.13499	2.40911
51	2.0771	-0.94564	-0.34463
52	0.1169	-3.12196	2.85139
53	-0.2964	0.90333	-1.25682
54	-2.4187	-1.80092	0.2813
55	-3.0375	1.50286	3.09061
56	1.5290	-2.97088	-1.54634
57	0.1530	-0.01951	0.14218
58	-0.9203	-1.64371	2.22836
59	2.6488	2.25934	-1.49084
60	-2.5165	-0.46549	-0.09963
61	-0.4344	2.26634	2.5801
62	0.8895	-1.4276	-1.00357
63	2.1702	1.22248	2.87771
64	-1.5633	-1.93941	0.04429
65	-0.7039	1.07472	-2.79157
66	2.1541	-2.80148	0.36066
67	2.8912	-0.28353	-2.43624
68	-0.3947	2.33779	0.50735
69		-0.91657	-3.00861
70			0.13762

Table 3c. New Polyphase Barkers			
	Length		
i	72	76	77
3	-0.1198	-0.2554	0.4842
4	-1.0806	-0.9594	1.3605
5	-1.9605	-0.8675	1.4647
6	-2.4183	-0.3549	1.3807
7	2.8421	-0.021	0.6939
8	2.3423	-0.9241	0.7884
9	2.1876	-1.5966	1.0706
10	1.0962	-1.2608	2.3907
11	0.7354	-1.667	3.1198
12	1.3757	3.0338	-2.9635
13	1.6034	1.3757	-1.6182
14	2.2493	-0.0373	-1.0878
15	2.3479	-1.1547	-0.4648
16	2.5616	-2.5654	1.9737
17	2.3693	1.6511	2.7293
18	-1.9032	1.1046	2.5286
19	-2.1234	1.0662	2.6055
20	-0.7022	-1.2648	2.2223
21	-1.3015	-1.4643	1.1757
22	1.8598	0.6193	1.482
23	0.878	0.7471	-1.444
24	-0.3462	-2.6102	-0.0035
25	-2.3613	2.7774	1.8144
26	2.9272	0.8379	1.4187
27	-0.2282	1.3649	0.0879
28	-1.751	2.8297	0.3141
29	2.0565	2.9632	-2.1991
30	0.4911	-0.847	-1.6148
31	-2.4045	-1.8794	2.151
32	-2.2363	1.97	1.4875
33	-2.5211	-0.1631	-0.5051
34	2.0985	-1.8844	0.0665
35	-0.5782	1.7583	-2.9933
36	-1.1035	-1.5173	0.3578
37	-2.297	3.1287	1.0729
38	2.3064	-0.7578	-1.5457
39	1.6868	-2.616	-2.5396

Table 3c, continued			
	Length		
i	72	76	77
40	-2.6224	0.7383	-0.1418
41	-1.8485	-0.3336	0.5553
42	-0.7337	0.2405	-2.3394
43	2.4022	0.7288	-2.6826
44	1.5541	0.3513	-0.5124
45	-2.8166	3.1116	0.8723
46	-2.3892	2.7773	-2.7094
47	-0.5668	2.9883	-3.0903
48	1.7357	-1.7581	-0.0609
49	3.0393	1.8147	0.5818
50	-0.6961	0.249	-2.3695
51	2.3551	2.1562	-0.2644
52	2.7485	-1.1582	-2.3454
53	-1.436	-2.0082	-2.1221
54	0.7501	-0.3944	1.7057
55	2.3623	-2.3461	-2.7964
56	-2.0554	0.2543	0.8776
57	2.2897	2.2108	2.9202
58	-0.3058	1.1797	-0.6545
59	2.8183	2.7053	0.8063
60	-0.0986	-1.7175	-1.7238
61	-2.9669	0.4087	2.6827
62	0.8018	-2.278	-0.5054
63	3.0655	-0.1442	-2.5864
64	-0.2771	2.9859	2.3057
65	2.5553	-0.3654	1.1339
66	-0.9381	2.3881	-1.7974
67	1.3781	-1.1393	1.3614
68	2.8031	0.4539	-0.1384
69	-0.7153	1.6986	-2.5185
70	2.4486	-2.2759	0.5594
71	-0.7853	0.2246	-2.7806
72	1.8288	2.2318	0.3097
73		-2.527	-2.8669
74		-0.5133	1.1971
75		2.0835	-1.0393
76		-1.6578	2.5406
77			-0.1159

III. If My Net Comes Back Empty, Are The Fish All Gone?

It is natural to ask why there is a dearth of larger polyphase Barkers in our results. As with previous efforts to find polyphase Barkers, it is not possible to determine for certain whether they were not found because they do not exist, or they were not found because the methods employed were not capable of finding them in the time alloted. Although this is not a question that can be answered for certain, evidence can be gathered and presented which suggests the answer to this question.

In an attempt to collect evidence of this kind, search yield, in terms of numbers of polyphase Barkers found, was determined for sizes from 10 to 52. As a first step, copious amounts of time and effort were spent in finding the number of polyphase Barkers for short lengths (below 20). The number of local optimizations needed to find all of the polyphase Barkers at these lengths were established. The effort was seen to be growing at an exponential rate. The amount of effort was then extrapolated from this lower length to lengths up to 52 chips. That effort was then expended, and the numbers tabulated; they are plotted in Figure 1.

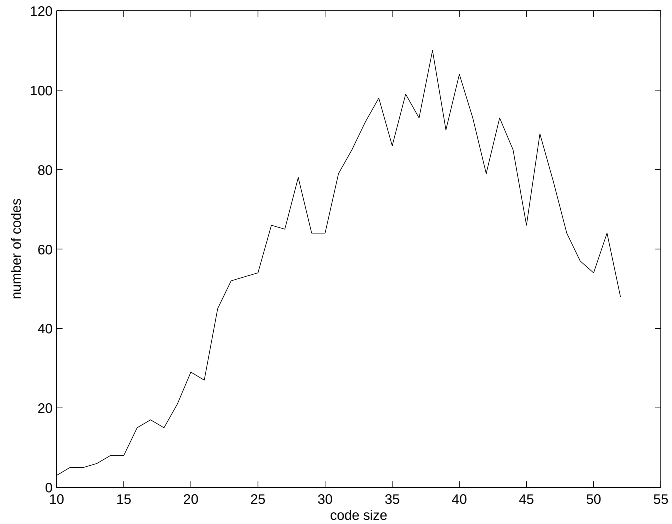

Fig. 1. Numbers of polyphase Barker codes as a function of code length

To see if the exponential increase in effort was sufficient to capture the number of polyphase Barker codes for lengths from 10 to 52, an additional test was done at length 52. In this test the program was run a large number of times over a period of several days on an 18 element Beowulf cluster. At the end of this time the program had executed 305 times. The exact number of times the routine was run had no particular significance other than the fact that it was a relatively large number. Each time the routine was run, a number of polyphase Barkers were found. Some were found every time, and some were found only a small percentage of the time. Figure 2 shows the results for the 48 codes found during the 305 trials.

Of the 305 times there were four codes found each of the 305 times. One code was found only 4 times and all the others were found 20 or more times. Interestingly, the number of codes found during this set of 305 trials was exactly the same as the number of codes found during the above-described exponentially increasing search. Similar tests were done for lengths 44 and 48. At each of these sizes the probabilities of finding individual codes followed a similar pattern, and we found no further codes beyond those found in the exponentially increasing search. This does not show that there are no more polyphase Barkers at that length. In fact a careful examination of Figure 2 suggests the possibility of a relatively small number of further polyphase Barkers at length 52. We do believe however that our search was relatively thorough at those lengths. It is the authors opinion that the numbers of poly-phase Barker codes not found during this search are not sufficient in number to effect the trend shown in Figure 1.

The shape of graph in Figure 1 appears to show that the number of polyphase Barker codes is rapidly decreasing after its peak at 38 chips. We believe that if we have made the case

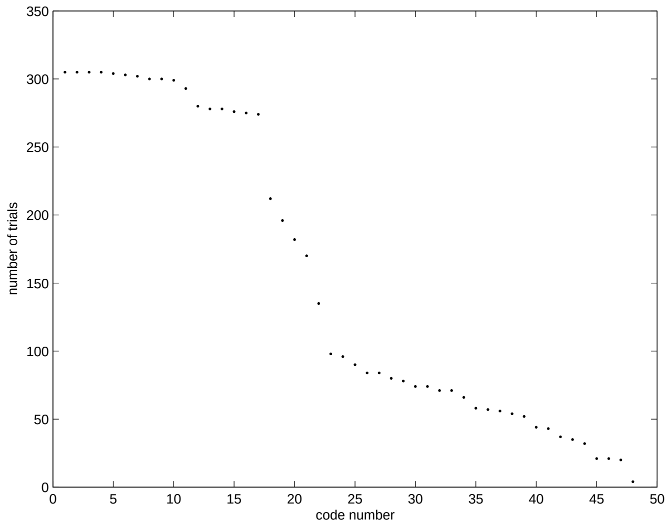

Fig. 2. Frequency of occurrence of polyphase Barkers of length 52

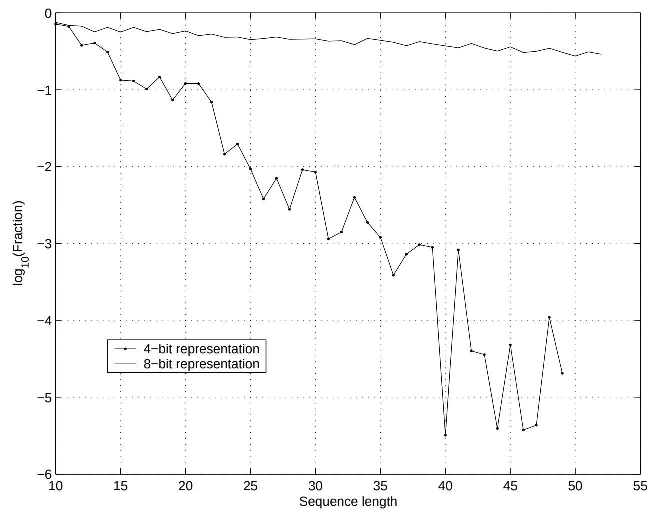

Fig. 3. Percentage of Sequences Remaining Barker for Random Phase Ramp, 6000 Trials

for having found most of the polyphase Barker codes in the 10 to 52 chip region, it is not unreasonable to believe that the polyphase Barker codes could run out at lengths not far beyond those found in this paper.

IV. Sensitivity to Doppler and Precision

Any pulse compression code will exhibit some sensitivity to Doppler and to the number of bits used to represent elements of the sequence. This section will show results from computations aimed at assessing each of these issues.

To assess sensitivity to quantization, all the Barker sequences found for lengths 10 to 52 were examined. For each of these lengths, each code was subjected to 6000 Monte Carlo trials in which two independent random phases were drawn from a uniform distribution. One phase was used to generate a unit-amplitude complex multiple for the sequence, and the second phase was used to generate a progressive ramp in phase across the sequence. Both of these operations preserve sidelobe level. However, the binary bit representation for the sequence generated was then truncated. Figure 3 shows the logarithm of the fraction of trials for which the sidelobes of these sequences obey the Barker constraint, as a function of length for 4-bit and 8-bit binary representation.

Figure 3 shows that the fraction decreases more slowly for the 8-bit representation than for the 4-bit representation, to about one quarter by length 52. The fraction with 4-bit representation drops below one out of every ten thousand cases by length 40. In each case the trend is for the proportion of polyphase Barkers to decrease log-linearly with length. This is in line with the authors' expectation that longer polyphase Barker codes can in general be expected to require larger alphabets. A more exhaustive study of this phenomenon is given in [9] and [11].

Doppler sensitivity was examined by applying a progressive phase ramp across each Barker sequences for length 10 to length 52. Phase ramps using increments of 0.25 and 0.5 degrees per chip were applied. Figure 4 shows the minimum,

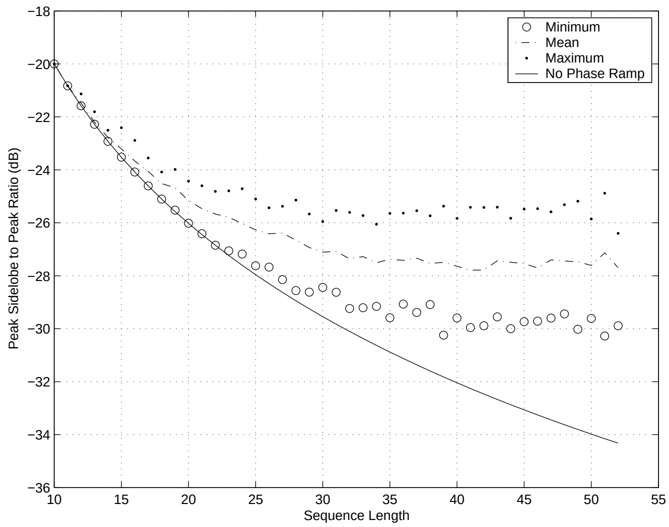

Fig. 4. Side-lobe levels of polyphase Barker codes after injecting 0.25 deg. of phase per chip

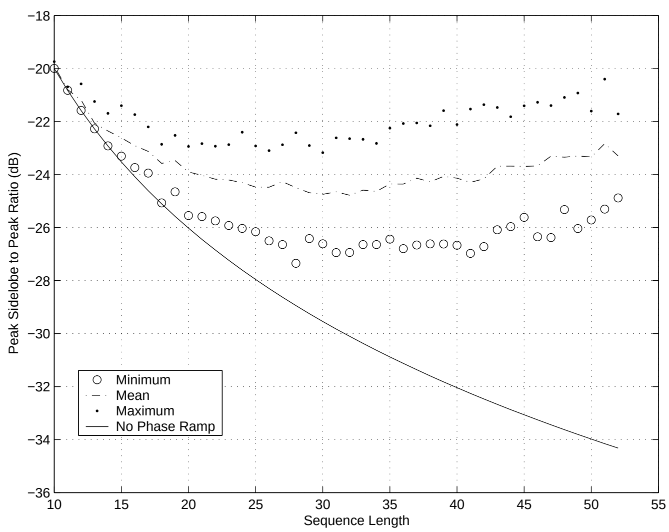

Fig. 5. Side-lobe levels of polyphase Barker codes after injecting 0.50 deg. of phase per chip

maximum and mean PSL observed for the sequences of each length with a phase ramp using an increment of 0.25 degrees. Figure 5 does the same for an increment of 0.50 degrees.

It should be noted that there is significant variability in the performance degradation due to Doppler effects, and in many of the cases exhibited the degraded sidelobes are better than can be expected for even the best of the match-filtered biphase codes.

V. Summary

In this paper we have extended the length for which polyphase Barker codes are known to exist to length 70. Further polyphase Barker codes were found at lengths 72, 76, and 77. In addition we extended the length for which globally minimum ISL codes are likely to have been found to length 80. We also exhibited evidence that the numbers of polyphase Barkers reach a peak in the late 30's and start decreasing at a fairly rapid rate. We do not claim that we have found all lengths for which there are polyphase Barkers. We do however believe that sometime in the near future, all the lengths for which polyphase Barkers exist will be found.

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