Conference

FROM BANACH SPACES TO FRAME THEORY AND APPLICATIONS

Celebrating the 65th Birthday of

Peter G. Casazza

May 20-22, 2010

Norbert Wiener Center
Department of Mathematics
University of Maryland
College Park, Maryland

SCHEDULE

	Thurs, May 20	Fri, May 21	Sat, May 22
8:00 – 8:45	Registration		
8:45 – 9:00	Welcome Speech		
9:00 – 10:00	John Benedetto	Bill Johnson	Vern Paulsen
10:00 – 10:30	Coffee Break	Coffee Break	Coffee Break
10:30 – 11:00	Götz Pfander	Stephen Dilworth	Ole Christensen
11:00 – 11:30	Kasso Okoudjou	Thomas Schlumprecht	Mark Lammers
11:30 – 12:00	Zeph Landau	Ed Odell	Matt Fickus
12:00 – 14:00	Lunch Break	Lunch Break	Lunch Break
14:00 – 15:00	Jelena Kovačević	Dick Kadison	Ingrid Daubechies
15:00 – 15:30	Birthday Cake	Coffee Break	Coffee Break
15:30 – 16:00	Joel Tropp	Eric Weber	Gitta Kutyniok
16:00 – 16:30	Akram Aldroubi	Darrin Speegle	Ali Pezeshki
16:30 – 17:00	Poster Session	Shidong Li	Yang Wang
18:00 – open end	Reception	Conference Dinner	Informal Dinner

Plenary Speakers

John Benedetto (University of Maryland) .... 9:00–10:00 a.m., Thursday, May 20
Sparse Representation: Two Case Studies

Ingrid Daubechies (Princeton University) .... 2:00–3:00 p.m., Saturday, May 22
TBA

Bill Johnson (Texas A&M University) .... 9:00–10:00 a.m., Friday, May 21
Dimension Reduction in Discrete Metric Geometry

Dick Kadison (University of Pennsylvania) .... 2:00–3:00 p.m., Friday, May 21
The Heisenberg–Von Neumann Puzzle

Nigel Kalton (University of Missouri) .... Conference Dinner, Friday, May 21

Jelena Kovačević (Carnegie Mellon University) .... 2:00–3:00 p.m., Thursday, May 20
Frames and Bunnies

Vern Paulsen (University of Houston) .... 9:00–10:00 a.m., Saturday, May 22
The Fourier Feichtinger Conjecture and the Kernels Conjecture

Invited Speakers

Akram Aldroubi (Vanderbilt University) .... 4:00–4:30 p.m., Thursday, May 20
Best Approximations by a Union on Subspaces

Ole Christensen (Technical University of Denmark) ..... 10:30–11:00 a.m., Friday, May 21
The Day Pete Entered Frame Theory

Stephen Dilworth (University of South Carolina) .... 10:30–11:00 a.m., Friday, May 21
Greedy Bases for Besov Spaces

Matthew Fickus (Air Force Institute of Technology) .... 11:30–12:00 a.m., Friday, May 21
Finding Nearby Unit Norm Tight Frames

Gitta Kutyniok (University of Osnabrück) .... 3:30–4:00 p.m., Friday, May 21
From Frames to Fusion Frames

Mark Lammers (U. North Carolina, Wilmington) .... 11:00–11:30 a.m., Saturday, May 22
Localization and Finite Frames

Zeph Landau (University of California, Berkeley) .... 11:30–12:00 a.m., Thursday, May 20
Redundancy for Infinite Frames

Shidong Li (San Francisco State University) .... 4:30–5:00 p.m., Thursday, May 21
Nonorthogonal Fusion Frames

Edward Odell (University of Texas) .... 11:30–12:00 a.m., Thursday, May 21
Embedding Banach Spaces

Kasso Okoudjou (University of Maryland) .... 11:00–11:30 a.m., Thursday, May 20
Invertibility of the Gabor Frame Operator on Wiener Amalgam Spaces

Ali Pezeshki (Colorado State University) .... 4:00–4:30 p.m., Friday, May 21
Robust Measurement Design for Detecting Sparse Signals

Götz Pfander (Jacobs University) .... 10:30–11:00 a.m., Thursday, May 20
Infinite Dimensional Restricted Invertibility and Applications to Gabor Frames

Thomas Schlumprecht (Texas A&M University) .... 11:00–11:30 a.m., Friday, May 21
Nonuniform Sampling and Recovery of Bandlimited Functions via Gaussians

Darrin Speegle (St. Louis University) .... 4:00–4:30 p.m., Friday, May 21
The Rado–Horn Theorem and Kadison–Singer

Joel Tropp (California Institute of Technology) .... 3:30–4:00 p.m., Thursday, May 20
The Sparsity Gap

Yang Wang (Michigan State University) .... 4:30–5:00 p.m., Friday, May 21
A New Approach to Analyzing Physiological Data

Eric Weber (Iowa State University) .... 3:30–4:00 p.m., Friday, May 21
Bases and Frames of Exponentials on the Cantor Set

Time : **4:00–4:30 p.m., Thursday, May 20**

Best Approximations by a Union on Subspaces

Akram Aldroubi

The problem of finding a union of subspaces that best approximations a set of data is a generalization of the problem of finding a single subspace that is closets to a finite sets of vectors. This problem has many applications is related to sparse approximations, compressed sensing, the generalized principle component analysis, and to the analysis of point sets in high dimension. It has many applications such as classification, motion tracking, and face recognition to name a few examples. In this talk, we will give describe the problem, its significance, and discuss some new results.

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Time : **9:00–10:00 a.m., Thursday, May 20**

Sparse Representation: Two Case Studies

John J. Benedetto

Two techniques are introduced where sparse representation plays a central role.

In the first, frame potential energy and penalty terms associated with Wiener amalgam spaces are used to provide early detection and classification of molecular photo-products of age-related macular degeneration, the leading cause of blindness among the elderly.

In the second, a mathematical theory a quantization, called Sparse Representation Quantization Procedure (SRQP), is developed. It is parallel to but different from Sigma-Delta modulation in the area of A/D non-linear coding in terms of low-bit signal approximants of uniformly sampled data.

The first technique is based on a collaboration between the Norbert Wiener Center and NIH, and it involves a host researchers including the following: Robert Bonner, Wojciech Czaja, Martin Ehler, Christopher Flake, and Matthew Hirn. The second technique is a collaboration with Onur Oktay.

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Time : **10:30–11:00 a.m., Friday, May 21**

The Day Pete Entered Frame Theory

Ole Christensen

To those who know Pete it will be no surprise to hear that he proved his first frame result on May 24, 1995 — the day he heard the definition of a frame. The talk will give a panoramic view of frame theory as it looked like in 1995, and how it developed after that. The emphasis will be on some of the areas where Pete has contributed, e.g., finite frames, Gabor theory, and duality. New results within these areas will be presented as well.

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$\boxed{\textit{Time}:\ \textbf{2:00–3:00 p.m., Saturday, May 22}}$

TBA

Ingrid Daubechies

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Time : **10:30–11:00 a.m., Friday, May 21**

Greedy Bases for Besov Spaces

Stephen Dilworth

This is joint work with Freeman, Odell, and Schlumprecht. A basis for a Banach space is *greedy* if the best n-term approximation to a vector is obtained (up to a multiplicative constant) by choosing the n largest coefficients. Konyagin and Temlyakov proved that a basis is greedy if and only if it is *unconditional* and *democratic*. Using this characterization we prove that the Banach space $(\oplus_{n=1}^{\infty} \ell_p^n)_{\ell_q}$, which is isomorphic to certain Besov spaces on the circle, has a greedy basis whenever $1 \leq p \leq \infty$ and $1 < q < \infty$. We also deduce from results of Bourgain, Casazza, Lindenstrauss, and Tzafriri on uniqueness of unconditional bases up to permutation that $(\oplus_{n=1}^{\infty} \ell_p^n)_{\ell_1}$ and $(\oplus_{n=1}^{\infty} \ell_p^n)_{c_0}$, with $1 \leq p < \infty$, do not have greedy bases. We also prove that the space $(\oplus_{n=1}^{\infty} \ell_p^n)_{\ell_q}$ has a 1-*greedy* basis (the corresponding isometric notion) if and only if $1 \leq p = q \leq \infty$.

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Time : **11:30–12:00 a.m., Friday, May 21**

Finding Nearby Unit Norm Tight Frames

Matthew Fickus

We consider the Paulsen problem: If a frame is close to being tight, and its frame elements are close to being equal norm, how close is it to an equal norm tight frame? Earlier work by Casazza and Bodmann provides one solution to this problem: by applying the inverse root of the frame operator, they find a nearby tight frame that isnearly equal norm; they then show the desired frame exists as the solution of a frame-energy-descending differential equation, provided the number of frame elements is relatively prime to the dimension of the underlying space. We provide an alternative solution to the Paulsen problem that differs from theirs in three significant respects. First, we begin by normalizing the frame elements themselves, rather than the frame operator, that is, we begin by finding a nearby unit norm frame that is nearly tight. Second, we produce a unit norm tight frame by descending the gradient of the frame potential, a discrete-time algorithm, rather than a continuous-time analytic method. Finally, though relative primeness remains an important consideration in our algorithm, we can show that regardless, it converges to a nearby unit norm tight frame. (This is joint work with Peter G. Casazza and Dustin G. Mixon.)

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Time : **9:00–10:00 a.m., Friday, May 21**

Dimension Reduction in Discrete Metric Geometry

William B. Johnson

In 1981 Lindenstrauss and I proved what has come to be known as the J-L lemma: if A is any set of n points in a Euclidean space, then A can be realized, with constant distortion, in $\mathbf{R}^d$ with $d \leq 1 + \log n$. This means that there is a function F from A into $\mathbf{R}^d$ so that for every pair of points x and y in A, $|x - y| \leq |F(x) - F(y)| \leq C|x - y|$. Moreover, F can be taken to be the restriction of a linear mapping from the span of A into $\mathbf{R}^d$ (this stronger version is called the linear J-L lemma). I'll review the proof of this old lemma and mention the application of it in the original paper Joram and I wrote, and then discuss more recent results on dimension reduction, including:

1. The theorem of Brinkman and Charikar that the J-L lemma fails in L_1. (The proof I'll outline is an argument due to Schechtman and me that further simplifies the beautiful argument of Lee and Naor.)

2. The result of Naor and mine that while there are spaces other than Hilbert spaces that satisfy the linear J-L lemma, any Banach space that satisfies the lemma must, in a certain sense, be extremely close to a Hilbert space.

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$\boxed{\textit{Time}:\ \textbf{2:00–3:00 p.m., Friday, May 21}}$

The Heisenberg–Von Neumann Puzzle

Richard Kadison

We discuss the Heisenberg relation, $QP - PQ = aI$, for some non-zero complex scalar a, from its origin in quantum physics, in the context of operator representations with special emphasis on a remarkable family of unbounded operators discovered by F.J. Murray and J. von Neumann. (Joint work with my student Zhe Liu.)

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Time : **2:00–3:00 p.m., Thursday, May 20**

Frames and Bunnies

Jelena Kovačević

I will talk about my experience the last 10 years in frame theory, most of which through collaboration and contact with Pete Casazza. I will touch upon finite-dimensional frames as well as the infinite-dimensional ones implementable by filter banks.

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Time : **3:30–4:00 p.m., Friday, May 21**

From Frames to Fusion Frames

Gitta Kutyniok

Frames have been a focus of study in the last two decades in applications where redundancy plays a vital and useful role. However, recently, a number of new applications have emerged which cannot be modeled naturally by one single frame system. They typically generally share a common property that requires distributed processing such as sensor networks.

In 2004, Pete and myself introduced the notion of frames of subspaces, which resulted later in 2006, joint with Shidong Li, in the framework of fusion frames. In this talk we review the development of fusion frames and show that they provide exactly the framework not only to model these applications but also to derive efficient and robust algorithms. Finally, some recent results will be discussed.

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Time : **11:00–11:30 a.m., Saturday, May 22**

Localization and Finite Frames

Mark Lammers

We consider the concepts of localization in time and frequency in the finite frame setting. We show that one can use these ideas to produce smooth alternate dual frames in the finite setting which may be used to reduce error in Sigma Delta quantization.

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Time : **11:30–12:00 a.m., Thursday, May 20**

Redundancy for Infinite Frames

Zeph Landau

Redundancy is the qualitative property which makes Hilbert space frames so useful in practice. However, developing a meaningful quantitative notion of redundancy for infinite frames has proven elusive. Recent joint work with the birthday boy and Radu Balan will be presented that establishes such a notion that holds for large classes of localized and Gabor frames.

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Time : **4:30–5:00 p.m., Thursday, May 21**

Nonorthogonal Fusion Frames

Shidong Li

Fusion Frame studies ways in which functions or signals from a set of subspaces can be combined coherently regardless how complicated subspaces are related. It has a deep root in data fusion applications for distributed systems such as sensor networks. Nevertheless, it has been seen that the fusion operation involves a fusion frame operator that is seldom sparse. While some applications can enjoy power constructions of Parseval fusion frames, a lot more distributed systems do not have the luxury for subspace selections, nor for subspace transformations/rotations. Non-orthogonal fusion frame extends fusion frames in which non-orthogonal projections become fundamental building blocks. We show that not only the (non-orthogonal) fusion frame operator can become sparse, it can also be made diagonal. Multi-fusion frames are also naturally introduced. As a result, the set of underlying subspaces no longer needs to be complete. Tight (non-orthogonal) fusion frames can be built based on one proper subspace. A notion of reflective sensing and its application are also introduced. Simple and natural implementation of the non-orthogonal fusion frames via pseudoframes for subspace will be discussed. More properties and applications will also be presented.

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Time : **11:30–12:00 a.m., Thursday, May 21**

Embedding Banach Spaces

Edward Odell

We will survey some embedding theorems in Banach spaces. Generally speaking the problem is given a separable space X, one wishes to embed it into a space Z with nicer structure which reflects the character of the space X. In particular we have the recent results.

Theorem A (Freeman, O., Schlumprecht). Let X^* be separable. Then X embeds into a space Y with Y^* isomorphic to ℓ^1.

Theorem B (Argyros, Freeman, Haydon, O., Raikoftsalis, Schlumprecht, Zisimopoulou). Let X^* be separable. If ℓ_1 does not embed into X^*, then X embeds into a space Y with the property that every bounded linear operator on Y is a compact perturbation of a scalar multiple of the identity.

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Time : **11:00–11:30 a.m., Thursday, May 20**

Invertibility of the Gabor Frame Operator on Wiener Amalgam Spaces

Kasso Okoudjou

In this talk, I'll introduce a generalization of Wiener's $1/f$ theorem to prove that for a Gabor frame with the generator in the Wiener amalgam space, the corresponding frame operator is invertible on this space. Therefore, for such a Gabor frame, the canonical dual inherits the properties of the generator. This talk is based on a joint work with I. Krishtal.

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Time : **9:00–10:00 a.m., Saturday, May 22**

The Fourier Feichtinger Conjecture and the Kernels Conjecture

Vern Paulsen

Thanks to the work of some unknown mathematician from Missouri, we now know that the Kadison–Singer conjecture is equivalent to the Feichtinger conjecture, i.e., to determining whether or not every frame can be partitioned into finitely many Riesz basic sequences. In this talk we will discuss two special cases of the Feichtinger conjecture where the frames are restricted to either arise from Fourier series or from reproducing kernels.

In the first case we consider a measurable subset A of the unit n-cube $[0,1]^n$. The functions $f_k(t) = \chi_A(t)\exp(2\pi k \cdot t)$ form a Parseval frame for $L^2(A)$, where $k = (k_1, ..., k_n) \in \mathbf{Z}^n$, $t = (t_1, ..., t_n) \in [0,1]^n$. Our work shows that if these "Fourier frames" can be partitioned into Riesz basic sequences, then each set in the partition can be taken to be a syndetic subset of $\mathbf{Z}^n$.

The second case that we discuss is when one has a reproducing kernel Hilbert space on a set X and a frame for this space arises as the set of normalized kernel functions at a sequence of points in X. Nikolskii was the first to study this special case of the Feichtinger conjecture and has shown that for some classical spaces, including the Hardy space and Bergman space of the unit disk, this "Feichtinger kernels conjecture" has a positive solution. In joint work with S. Lata, we have shown that the full Feichtinger conjecture is equivalent to solving the Feichtinger kernels conjecture for all de Branges spaces.

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Time : **4:00–4:30 p.m., Friday, May 21**

Robust Measurement Design for Detecting Sparse Signals

Ali Pezeshki

Detecting a sparse signal in noise is fundamentally different from reconstructing a sparse signal, as the objective is to optimize a detection performance criterion rather than to find the sparsest signal that satisfies a linear observation equation. In this paper, we consider the design of low dimensional (compressive) measurement matrices for detecting sparse signals in white Gaussian noise. We use a lexicographic optimization approach to maximize the worst-case signal-to-noise ratio (SNR), and to derive an expression for it as a function of the signal dimension and the number of measurements. Equiangular uniform tight frames and Grassmannian packings are central to our designs.

This is joint work with Ramin Zahedi and Edwin K. P. Chong.

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Time : **10:30–11:00 a.m., Thursday, May 20**

Infinite Dimensional Restricted Invertibility and Applications to Gabor Frames

Götz Pfander

The Bourgain–Tzafriri Restricted Invertibility Theorem states conditions under which a Riesz bases can be extracted from a possibly overcomplete system of vectors in finite dimensional spaces. We extend the result to vector dictionaries in infinite dimensional Hilbert spaces using techniques developed in the theory of localized frames, and we state applications to Gabor systems.

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Time : **11:00–11:30 a.m., Friday, May 21**

Nonuniform Sampling and Recovery of Bandlimited Functions viaGaussians

Thomas Schlumprecht

We approximate, via interpolation,multi dimensional bandlimited functions via non uniform sampling by shifts ofGaussian Radial-Basis functions.

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$\boxed{Time: \textbf{4:00–4:30 p.m., Friday, May 21}}$

The Rado–Horn Theorem and Kadison–Singer

Darrin Speegle

In this talk, I will present an overview of how the Rado–Horn Theorem can be used to get partial results on the Kadison–Singer Problem. In one of its forms, the Kadison–Singer Problem asks whether every Bessel sequence in a Hilbert space which is bounded below in norm can be partitioned into finitely many Riesz sequences. The Rado–Horn Theorem tells us, among other things, that every such Bessel sequence can be partitioned into linearly independent sets. In this talk, we will discuss this and other applications of the Rado–Horn Theorem to partitioning sequences in Hilbert spaces. We will also discuss how having a partition into linearly independent sets can be exploited to partition Bessel sequences into ℓ_2 linearly independent sets.

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Time : **3:30–4:00 p.m., Thursday, May 20**

The Sparsity Gap

Joel Tropp

In an incoherent dictionary, most signals that admit a sparse representation admit a unique sparse representation. In other words, there is no way to express the signal without using strictly more atoms. This work demonstrates that sparse signals typically enjoy a higher privilege: any other sparse representation of the signal requires a lot of additional atoms. This finding has an impact that is primarily philosophical; if one has in hand a sparse representation of a signal over an incoherent dictionary, the theorem confers on this particular representation a degree of legitimacy.

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$\boxed{\textit{Time}: \textbf{4:30–5:00 p.m., Friday, May 21}}$

A New Approach to Analyzing Physiological Data

Yang Wang

We propose a new method for analyzing and classifying physiological data. This method is based on an alternative empirical mode decomposition (EMD) algorithm called iterative filtering. Classifiers can be designed after the decomposition using support vector machines. We illustrate the effectiveness of this approach from the analysis of cardiac interbeat time series.

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Time : **3:30–4:00 p.m., Friday, May 21**

Bases and Frames of Exponentials on the Cantor Set

Eric Weber

Given a singular probability measure on the real line, when is it possible to have a basis or frame of exponentials? For the so-called Cantor-4 set, described as numbers with digits 0 and 2 in a base 4 expansion, Jorgensen and Pedersen constructed an explicit orthonormal basis for the Hausdorff measure supported on that set. For the usual Cantor-3 set, those numbers with digits of 0 and 2 in a base 3 expansion, Jorgensen and Pedersen showed that there cannot be an orthonormal basis of exponentials for the Hausdorff measure supported on it. We discuss possible algorithms, and impediments, to construct non-orthogonal bases and frames for the Cantor-3 set.

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