

Limited Tax Rebates, Tax
Arbitrage and
Equilibrium in a Multi-period
Financial Market with
General Taxes Structures and
Transaction Costs

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Arbitrage and equilibrium based asset pricing in finite economies standard textbook models.

Complete and Incomplete market models well understood.

Incomplete market models can be rationalized as complete market models with transaction costs in trading assets. This theory is reasonably well-developed.

Theory of asset markets with taxation is less developed – the reasons are that tax policies are often either very complex, or relatively trivial.

The trivial case is where assets are taxed equally – in a perfect arbitrage case the net profit of the replicating strategy is zero. Thus taxes have no implication for relative asset prices.

The complex case occurs in the situations where:

1. differential taxation of assets – eg, debt vs equity; assets taxed in different tax regimes. This theory is less developed as it requires careful modeling of the tax code and analysis of equilibrium asset prices and asset allocations is non-trivial. (Examples abound in corporate finance.)
2. Dynamic Taxation codes eg. capital gains taxation, that varies according to type of asset and realization.

This Paper

We construct a multi-period, general equilibrium model with transaction costs (to capture endogenous incomplete markets); and general taxation functions.

We include consumers, productive firms, asset transaction firms (brokers or intermediaries) and a government.

There is a finite time, filtration set-up. This is mathematically easy so that we can concentrate on some subtle economic issues.

We assume finite commodities and services at each event. They are traded on competitive markets – the transaction costs occur only for financial assets.

Jin-Milne (1999) proves existence with TC but no government and taxes. Existence implies asset prices are arbitrage free modulo TC. The trick in proving existence for such an economy is an old one in Econ Theory – double the dimension of the asset space so that there are bought and sold assets and associated prices. The spread is explained by the marginal TC.

Note: this model *cannot* handle non-competitive liquidity issues – the model is perfectly competitive.

It is known that introducing asset taxation can create problems with tax arbitrage (see Schaefer (1982), Dammon and Green (1987) for two date models.

Jones and Milne (1992) show in a two date model that the problem relates to incomplete modeling of the government's ability to fund unlimited tax arbitrages. More realistically,

government tax laws bound tax arbitrage claims long before they bankrupt govts!

Introducing those elements of the tax code enables the use of the standard type of existence proof. Feasible asset trades are bounded for the economy.

This paper extends that paper in several directions (many commodities, many periods, general tax functions that handle income and capital gains etc) and provides a consistent framework for a competitive asset economy that deals with TC and Tax frictions.

I will sketch the basic idea of the proof – and the key tricks to show existence. The technical details are standard in this literature – fixed point theorem, Maximum theorem, upper-hemi continuity etc.

Here are the key assumptions – later we will discuss weaker assumptions and techniques for a more general model.

A. We assume that all agents have preferences that are standard in economics. This avoids two major problems:

1. With incomplete markets and certain types of taxation systems firms may not have a well-defined profit objective. There are various ways around this – we assume the existence of a utility function.

2. We assume a utility function for the govt. This is a gross simplification – but avoids objectives that could have discontinuous responses.

B. TC are generated by convex technology – this rules out fixed costs etc – more later.

C. Tax functions are convex and bounded – this is reasonable for most taxes – there are examples of tax and social welfare systems with non-convex elements – more later.

The strategy of the proof and the key tricks:

1. We require the set of feasible to asset and commodity trades to be closed and bounded. The key here are two elements:

A. TC always are always positive, although could be small – trades use resources.

B. the govt has bounded resources and is part of the economy.

These two assumptions plus standard technical assumptions imply compact feasible sets in the finite space.

2. Given compactness of feasible sets, standard preferences, convex production sets etc, then a variation on a standard proof gives us existence of a competitive equilibrium.

3. As a corollary there cannot be buying and selling asset prices such that there are unbounded arbitrage opportunities on asset trades. This would contradict the existence of an equilibrium.

We know this result from much previous work, which assumes the absence of arbitrage and deduces restrictions on buying/selling asset prices. Here it drops out from the proof of the existence of an equilibrium.

Also observe that market incompleteness is determined endogenously – it is the absence of activity in an asset market. In this model the existence of all asset markets are assumed – it is activity that is the issue. (More on this later).

Generalizations:

There are a number of assumptions that can be weakened to allow for a more realistic model. The most significant are:

1. Utility functions for brokers and firms – some such assumption is unavoidable. You can weaken the assumptions on preferences – see Kelsey-Milne (1996).
2. Convex TC technology. More realistically we can allow some non-convexity in brokers' technology sets, but assume large numbers of brokers – then standard GE tricks can give us existence. (see Jin and Milne (1999) for discussion
3. The TC technology can be modified easily to allow for portfolio restrictions and personal TC on consumers. The trick is to redefine the assets to be personalized. (Milne-Neave(2003).

4. The taxation system allows for contingent taxation. This allows for random auditing and other devices for taxing tax avoidance schemes.
5. The model assumes that the govt pre-commits to a taxation regime. If the govt changes the regime, we can accommodate that as a rationally anticipated part of the decision tree. If the change is not anticipated, the equilibrium will be changed – predictions will depend on the new equilibrium.

Efficiency and Uniqueness:

1. The equilibrium will not necessarily be unique – this is an old problem in GE. Simple GE models in the finance literature assume this problem away by making very strong assumptions.
2. The equilibria will not be efficient in general – incompleteness and taxation. But, taxation may be aimed to help overcome inefficiencies. Example: incompleteness can be simulated by taxation (subsidy) functions. Huge literature in Economics on this - there seems to be little interaction with the finance literature.

Characterizations of Equilibria:

1. Many TC characterizations – some based on no arbitrage and martingale pricing – non-unique martingales. Examples of bounds on derivative pricing.
2. Some based on utility, equilibrium models – these usually tightly specified in terms of utility etc.
3. Comparative static models. This is a tricky area. See Boyle-Wang (MF 2001). Introducing “new” assets can have implications for other asset prices. Well-known result in Econ Theory.

Taxation (some general observations):

1. A few results known – mainly in Corporate Finance. MM and Tax.
2. Derivative pricing largely ignores tax as common taxes it the portfolio and taxation as no implication for relative prices.
3. But differential taxation is crucial to understanding the use of derivatives for tax reduction. I have seen little in this area on pricing and positions.

Taxation and TC (some quick comments):

1. The main literature on this is in Econ dealing with missing (inactive) markets and tax/subsidy schemes to improve efficiency.

Example: Bob Shiller on introducing income contingent assets. Public Economics literature argues that markets are missing for good reasons – want govt intervention.

2,. Much Corporate Fin and Tax is loose on completeness and incompleteness of asset markets – this matters, as Fisher Thm often does not hold.