Lévy driven fixed income models

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Lévy term structure models

The Lévy forward rate model (HJM type)

$$f(t,T) = f(0,T) + \int_0^t \alpha(s,T)ds - \int_0^t \sigma(s,T)dL_s$$

The Lévy forward process model

$$F(t,T_j^*,T_{j-1}^*) = F(0,T_j^*,T_{j-1}^*)\exp\left(\int_0^t \lambda(s,T_j^*)dL_s^{T_{j-1}^*}\right)$$

The Lévy Libor or market model

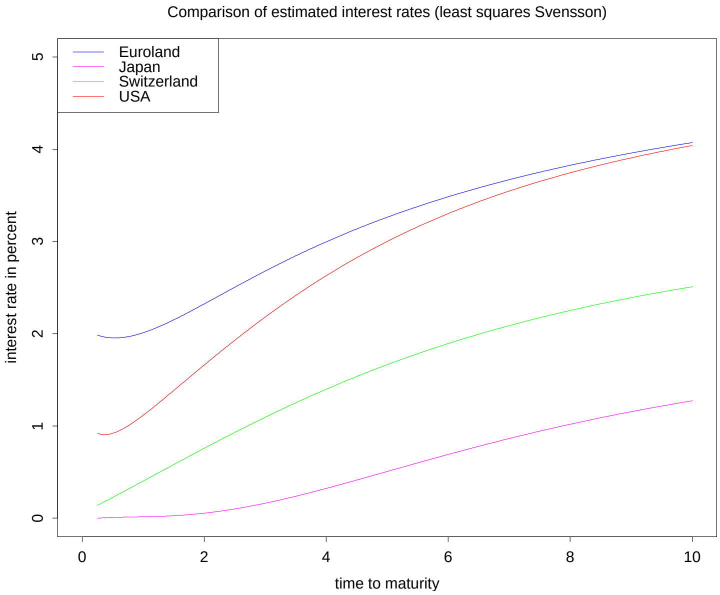
$$L(t,T_j^*) = L(0,T_j^*)\exp\left(\int_0^t \lambda(s,T_j^*)dL_s^{T_{j-1}^*}\right)$$

$$1 + \delta L(t,T_j^*) = F(t,T_j^*,T_{j-1}^*)$$

Extensions of the Lévy market model

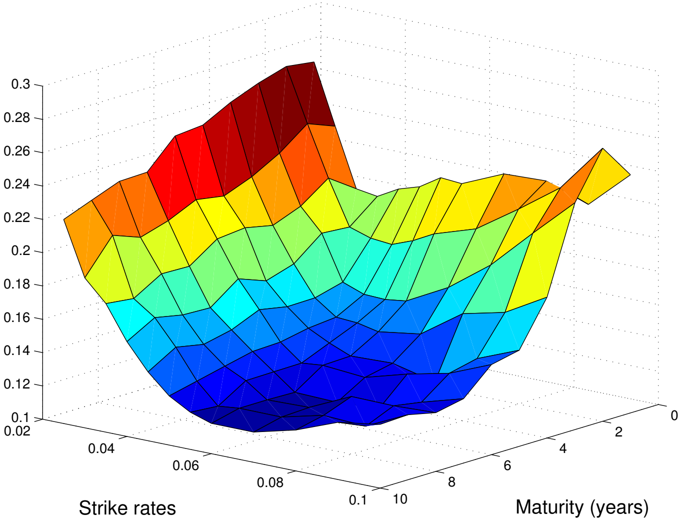
Lévy market model

- Multi-currency setting (Eb–Koval (2006))
- Credit risk model (Eb–Kluge–Schönbucher (2006))
- Swap rate model (Eb–Liinev (2006))
- Duality principle (Eb–Kluge–Papapantoleon (2006))



Termstructure, February 17, 2004

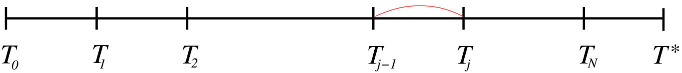
Caplet market data



Euro caplet implied volatility surface on February 19, 2002

Libor rates in a cross currency setting

Discrete tenor structure $T_0 < T_1 < \cdots < T_n < T_{n+1} = T^*$
Accrual periods $\delta = T_{j+1} - T_j$



$(m+1)$ markets $\quad i = 0, \dots, m$
0 = domestic market

Want to model the dynamics of the Libor rate $L^i(t, T_{j-1})$ which applies to time period $[T_{j-1}, T_j]$ in market i ($i = 0, \dots, m$)

We target at the form

$$L^i(t, T_{j-1}) = L^i(0, T_{j-1}) \exp\left(\int_0^t \lambda^i(s, T_{j-1}) dL_s^{i,T_j} \right)$$

The driving process

$L^{0,T^*} = (L_1^{0,T^*}, \dots, L_d^{0,T^*})$ is a d-dimensional time-inhomogeneous Lévy process. The law of L_t^{0,T^*} is given by

$$\mathbb{E}[\exp(\mathrm{i}u^\top L_t^{0,T^*})] = \exp \int_0^t \theta_s^{0,T^*}(\mathrm{i}u)\,\mathrm{d}s \qquad \text{with}$$

$$\theta_s^{0,T^*}(z) = z^\top b_s^{0,T^*} + \frac{1}{2} z^\top C_s z + \int_{\mathbb{R}^d} \left(e^{z^\top x} - 1 - z^\top x\right) \lambda_s^{0,T^*}(\mathrm{d}x),$$

where $b_t^{0,T^*} \in \mathbb{R}^d$, C_s is a symmetric nonnegative-definite $d \times d$-matrix and λ_s^{0,T^*} is a Lévy measure.

Integrability: $\displaystyle\int_0^{T^*} \left(|b_s^{0,T^*}| + ||C_s|| + \int_{\{|x|\le 1\}} |x|^2 \lambda_s^{0,T^*}(\mathrm{d}x)\right) \mathrm{d}s < \infty$

$$\int_0^{T^*} \int_{\{|x|>1\}} \exp(u^\top x) \lambda_s^{0,T^*}(\mathrm{d}x)\,\mathrm{d}s < \infty \qquad (u \in [-M, M]^d)$$

Description in terms of modern stochastic analysis

$L^{0,T^*} = (L_t^{0,T^*})$ is a special semimartingale with canonical representation

$$L_t^{0,T^*} = \int_0^t b_s^{0,T^*}\, \mathrm{d}s + \int_0^t c_s\, \mathrm{d}W_s^{0,T^*} + \int_0^t \int_{\mathbb{R}^d} x(\mu - \nu_{0,T^*})(\mathrm{d}s, \mathrm{d}x)$$

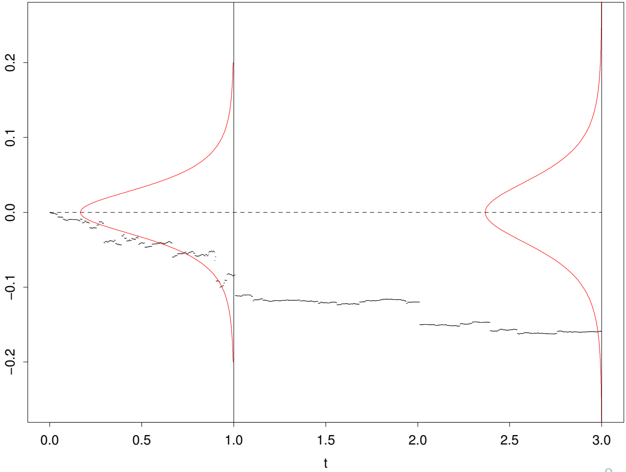
(W_t^{0,T^*}) is a $\mathbb{P}^{0,T^*}$-standard Brownian motion with values in $\mathbb{R}^d$

c_t is a measurable version of the square root of C_t
μ the random measure of jumps of (L_t^{0,T^*})

$\nu_{0,T^*}(\mathrm{d}s, \mathrm{d}x) = \lambda_s^{0,T^*}(\mathrm{d}x)\, \mathrm{d}s$ is the $\mathbb{P}^{0,T^*}$-compensator of μ
(L_t^{0,T^*}) is also called a process with *independent increments* and *absolutely continuous characteristics (PIIAC)*.

Simulation of a Lévy process

NIG(10,0,0.100,0) on [0,1]
NIG(10,0,0.025,0) on [1,3]



The foreign forward exchange rate for date T^* (1)

Assumption

(FXR.1): *For every market* $i \in \{0, \ldots, m\}$ *there are strictly decreasing and strictly positive zero-coupon bond prices* $B^i(0,T_j)(j = 0, \ldots, N+1)$ *and for every foreign economy* $i \in \{1, \ldots, m\}$ *there are spot exchange rates* $X^i(0)$ *given.*

Consequently the initial foreign forward exchange rate corresponding to T^* *is*

$$F_{X^i}(0,T^*) = \frac{B^i(0,T^*)X^i(0)}{B^0(0,T^*)}$$

The foreign forward exchange rate for date T^* (2)

Assumption

(FXR.2): *For every foreign market* $i \in \{1,\ldots,m\}$ *there is a continuous deterministic function* $\xi^i(\cdot,T^*) : [0,T^*] \to \mathbb{R}^d_+$.

For every coordinate $1 \le k \le d$ *we assume*

$$(\xi^i(s,T^*))_k \le \overline{M} \quad (s \in [0,T^*],\ 1 \le i \le m)$$

where $\overline{M} < \dfrac{M}{N+2}$.

The foreign forward exchange rate for date T^* (3)

Assumption

(FXR.3): *For every* $i \in \{1, \dots, m\}$ *the foreign forward exchange rate for date* T^* *is given by*

$$F_{X^i}(t,T^*) = F_{X^i}(0,T^*) \exp\left(\int_0^t \gamma^i(s,T^*)\,\mathrm{d}s + \int_0^t \xi^i(s,T^*)^\top \,\mathrm{d}L_s^{0,T^*} \right)$$

where

$$\begin{aligned} \gamma^i(s,T^*) &= -\xi^i(s,T^*)^\top b_s^{0,T^*} - \frac{1}{2}\left|\xi^i(s,T^*)^\top c_s\right|^2 \\ &\quad - \int_{\mathbb{R}^d} \left(e^{\xi^i(s,T^*)^\top x} - 1 - \xi^i(s,T^*)^\top x \right) \lambda_s^{0,T^*}(\mathrm{d}x) \end{aligned}$$

Equivalently

$$\begin{aligned} F_{X^i}(t,T^*) &= F_{X^i}(0,T^*)\mathcal{E}_t\Bigg(\int_0^\cdot \xi^i(s,T^*)^\top c_s \,\mathrm{d}W_s^{0,T^*} \\ &\quad + \int_0^\cdot \int_{\mathbb{R}^d} \left(\exp\left(\xi^i(s,T^*)^\top x\right) - 1 \right)(\mu - \nu_{0,T^*})(\mathrm{d}s,\mathrm{d}x) \Bigg) \end{aligned}$$

The foreign forward exchange rate for date T^* (4)

Consequences: $F_{X^i}(\cdot, T^*)$ is a $\mathbb{P}^{0,T^*}$-martingale

$$E_{\mathbb{P}^{0,T^*}}\left[\frac{F_{X^i}(t,T^*)}{F_{X^i}(0,T^*)}\right] = 1$$

Define

$$\left.\frac{d\mathbb{P}^{i,T^*}}{d\mathbb{P}^{0,T^*}}\right|_{\mathcal{F}_t} = \frac{F_{X^i}(t,T^*)}{F_{X^i}(0,T^*)}$$

By Girsanov's theorem we get a $\mathbb{P}^{i,T^*}$-standard Brownian motion

$$W_t^{i,T^*} = W_t^{0,T^*} - \int_0^t c_s \xi^i(s,T^*)\, ds$$

and a $\mathbb{P}^{i,T^*}$-compensator

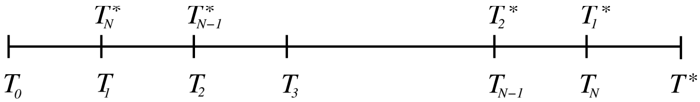
$$\nu_{i,T^*}(dt, dx) = \exp(\xi^i(t,T^*)^\top x)\nu_{0,T^*}(dt, dx)$$

The Lévy Libor model

Eberlein–Özkan (2005)

Tenor structure $T_0 < T_1 < \cdots < T_N < T_{N+1} = T^*$

with $T_{j+1} - T_j = \delta$, set $T_j^* = T^* - j\delta$ for $j = 1, \ldots, N$



Assumptions

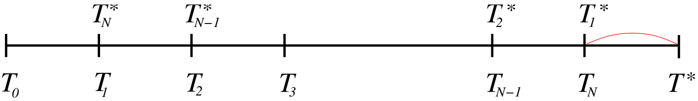
(CLM.1): *For every market i and every maturity T_j there is a bounded deterministic function $\lambda^i(\cdot,T_j)$, which represents the volatility of the forward Libor rate process $L^i(\cdot,T_j)$ in market i.*

(CLM.2): *The initial term structure of forward Libor rates in market i is given by*

$$L^i(0,T_j) = \frac{1}{\delta}\left(\frac{B^i(0,T_j)}{B^i(0,T_j+\delta)} - 1\right)$$

Backward Induction (1)

Given a stochastic basis $(\Omega, \mathcal{F}_{T^*}, \mathbb{P}^{0,T^*}, (\mathcal{F}_t)_{0\leq t\leq T^*})$



We postulate that under $\mathbb{P}^{i,T^*}$

$$L^i(t, T_1^*) = L^i(0, T_1^*) \exp\left(\int_0^t \lambda^i(s, T_1^*)\, dL_s^{i,T^*}\right)$$

where

$$L_t^{i,T^*} = \int_0^t b_s^{i,T^*}\, ds + \int_0^t c_s\, dW_s^{i,T^*} + \int_0^t \int_{\mathbb{R}^d} x(\mu - \nu_{i,T^*})(ds, dx)$$

with W^{i,T^*} and ν_{i,T^*} as given before.

Backward Induction (2)

In order to make $L^i(t,T_1^*)$ a $\mathbb{P}^{i,T^*}$-martingale, choose the drift characteristic (b_s^{i,T^*}) s.t.

$$\begin{aligned}\int_0^t \lambda^i(s,T_1^*)b_s^{i,T^*}\,\mathrm{d}s &= -\frac{1}{2}\int_0^t |\lambda^i(s,T_1^*)c_s|^2\,\mathrm{d}s \\ &\quad - \int_0^t\int_{\mathbb{R}^d}\left(e^{\lambda^i(s,T_1^*)x} - 1 - \lambda^i(s,T_1^*)x\right)\nu_{i,T^*}(\mathrm{d}s,\mathrm{d}x)\end{aligned}$$

Transform $L^i(t,T_1^*)$ in a stochastic exponential

$$L^i(t,T_1^*) = L^i(0,T_1^*)\mathcal{E}_t(H^i(\cdot,T_1^*))$$

where

$$H^i(t,T_1^*) = \int_0^t \lambda^i(s,T_1^*)c_s\,\mathrm{d}W_s^{i,T^*} + \int_0^t\int_{\mathbb{R}^d}\left(e^{\lambda^i(s,T_1^*)x} - 1\right)(\mu - \nu_{i,T^*})(\mathrm{d}s,\mathrm{d}x)$$

Backward Induction (3)

Equivalently

$$\begin{aligned} \mathrm{d}L^i(t,T_1^*) &= L^i(t-,T_1^*)\Big(\lambda^i(t,T_1^*)c_t\,\mathrm{d}W_t^{i,T^*} \\ &\quad + \int_{\mathbb{R}^d}\left(e^{\lambda^i(t,T_1^*)x}-1\right)(\mu-\nu_{i,T^*})(\mathrm{d}t,\mathrm{d}x)\Big) \end{aligned}$$

with initial condition

$$L^i(0,T_1^*) = \frac{1}{\delta}\left(\frac{B^i(0,T_1^*)}{B^i(0,T^*)}-1\right)$$

Backward Induction (4)

Recall $F_{B^i}(t,T_1^*,T^*) = 1 + \delta L^i(t, T_1^*)$, therefore,

$$\begin{aligned} dF_{B^i}(t,T_1^*,T^*) &= \delta\, dL^i(t,T_1^*) \\ &= F_{B^i}(t-,T_1^*,T^*)\Bigg(\underbrace{\frac{\delta L^i(t-,T_1^*)}{1+\delta L^i(t-,T_1^*)}\lambda^i(t,T_1^*)\,c_t}_{=\alpha^i(t,T_1^*,T^*)}\, dW_t^{i,T^*} \\ &\quad + \int_{\mathbb{R}^d} \underbrace{\frac{\delta L^i(t-,T_1^*)}{1+\delta L^i(t-,T_1^*)}\left(e^{\lambda^i(t,T_1^*)x}-1\right)}_{=\beta^i(t,x,T_1^*,T^*)-1}(\mu-\nu_{i,T^*})(dt,dx)\Bigg) \end{aligned}$$

Define the forward martingale measures associated with T_1^*

$$\frac{d\mathbb{P}^{i,T_1^*}}{d\mathbb{P}^{i,T^*}} = \mathcal{E}_{T_1^*}(M^{i,1}) \qquad \text{where}$$

$$M_t^{i,1} = \int_0^t \alpha^i(s,T_1^*,T^*)c_s\, dW_s^{i,T^*} + \int_0^t\int_{\mathbb{R}^d}\left(\beta^i(s,x,T_1^*,T^*)-1\right)(\mu-\nu_{i,T^*})(ds,dx)$$

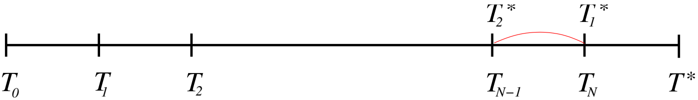
Backward Induction (5)

Then $$W_t^{i,T_1^*} = W_t^{i,T^*} - \int_0^t \alpha^i(s, T_1^*, T^*) c_s \, \mathrm{d}s$$

is the forward Brownian motion for date T_1^* and

$$\nu_{i,T_1^*}(\mathrm{d}t, \mathrm{d}x) = \beta^i(t, x, T_1^*, T^*) \nu_{i,T^*}(\mathrm{d}t, \mathrm{d}x)$$ is the $\mathbb{P}^{i,T_1^*}$-compensator for μ.

Second step



We postulate that under $\mathbb{P}^{i,T_1^*}$

$$L^i(t, T_2^*) = L^i(0, T_2^*) \exp\left(\int_0^t \lambda^i(s, T_2^*) \, \mathrm{d}L_s^{i,T_1^*} \right)$$ where

$$L_t^{i,T_1^*} = \int_0^t b_s^{i,T_1^*} \, \mathrm{d}s + \int_0^t c_s \, \mathrm{d}W_s^{i,T_1^*} + \int_0^t \int_{\mathbb{R}^d} x(\mu - \nu_{i,T_1^*})(\mathrm{d}s, \mathrm{d}x)$$

Backward Induction (6)

Second measure change

$$\frac{d\mathbb{P}^{i,T_2^*}}{d\mathbb{P}^{i,T_1^*}} = \mathcal{E}_{T_2^*}(M^{i,2})$$

where

$$M_t^{i,2} = \int_0^t \alpha^i(s,T_2^*,T_1^*)c_s \, dW_s^{i,T_1^*} + \int_0^t \int_{\mathbb{R}^d} \left(\beta^i(s,x,T_2^*,T_1^*) - 1\right)(\mu - \nu_{i,T_1^*})(ds,dx)$$

This way we get for each time point T_j^* in the tenor structure a Libor rate process which is under the forward martingale measure $\mathbb{P}^{i,T_{j-1}^*}$ of the form

$$L^i(t,T_j^*) = L^i(0,T_j^*) \exp\left(\int_0^t \lambda^i(s,T_j^*) \, dL_s^{i,T_{j-1}^*}\right)$$

Alternative Backward Induction (1)

Postulate

$$1+\delta L^i(t,T_1^*) = (1+\delta L^i(0,T_1^*)) \exp\left(\int_0^t \lambda^i(s,T_1^*)\,\mathrm{d}L_s^{i,T^*}\right)$$

equivalently

$$F_{B^i}(t,T_1^*,T^*) = F_{B^i}(0,T_1^*,T^*) \exp\left(\int_0^t \lambda^i(s,T_1^*)\,\mathrm{d}L_s^{i,T^*}\right)$$

In differential form

$$\begin{aligned}\mathrm{d}F_{B^i}(t,T_1^*,T^*) \;=\; F_{B^i}(t-,T_1^*,T^*)\Big(&\lambda^i(t,T_1^*)c_t\,\mathrm{d}W_t^{i,T^*} \\ &+\int_{\mathbb{R}^d}\left(\mathrm{e}^{\lambda^i(t,T_1^*)x}-1\right)(\mu-\nu_{i,T^*})(\mathrm{d}t,\mathrm{d}x)\Big)\end{aligned}$$

Alternative Backward Induction (2)

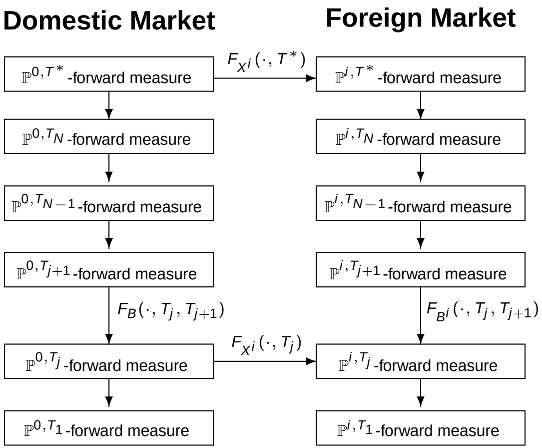
Define the forward martingale measures associated with T_1^*

$$\frac{d\mathbb{P}^{i,T_1^*}}{d\mathbb{P}^{i,T^*}} = \mathcal{E}_{T_1^*}(\widetilde{M}^{i,1})$$

where

$$\widetilde{M}_t^{i,1} = \int_0^t \lambda^i(s,T_1^*)c_s \, dW_s^{i,T^*} + \int_0^t \int_{\mathbb{R}^d} \left(e^{\lambda^i(s,T_1^*)x} - 1\right)(\mu - \nu_{i,T^*})(ds,dx).$$

Cross-currency Lévy market model



Relationship between domestic and foreign fixed income markets in a discrete-tenor framework.

Relationship between the domestic and the foreign market

The forward exchange rates in the i-th foreign market are related by

$$F_{X^i}(t,T_j) = F_{X^i}(t,T_{j+1})\frac{F_{B^i}(t,T_j,T_{j+1})}{F_{B^0}(t,T_j,T_{j+1})}$$

From this one gets the dynamics of $F_{X^i}(t,T_j)$

$$\frac{\mathrm{d}F_{X^i}(t,T_j)}{\mathrm{d}F_{X^i}(t-,T_j)} = \zeta^i(t,T_j,T_{j+1})\,\mathrm{d}W_t^{0,T_j} + \int_{\mathbb{R}^d}(\overline{\zeta}^i(t,x,T_j,T_{j+1})-1)(\mu-\nu_{0,T_j})(\mathrm{d}t,\mathrm{d}x)$$

where the coefficients are given recursively

$$\begin{aligned}\zeta^i(t,T_j,T_{j+1}) &= \alpha^i(t,T_j,T_{j+1}) - \alpha^0(t,T_j,T_{j+1}) + \zeta^i(t,T_{j+1},T_{j+2})\\ \overline{\zeta}^i(t,x,T_j,T_{j+1}) &= \frac{\beta^i(t,x,T_j,T_{j+1})}{\beta^0(t,x,T_j,T_{j+1})}\,\overline{\zeta}^i(t,x,T_{j+1},T_{j+2})\end{aligned}$$

Pricing cross-currency derivates (1)

Foreign forward caps and floors

$$\delta X [L^i(T_{j-1}, T_{j-1}) - K^i]^+$$

Time-0-value of a foreign T_N-maturity cap

$$\mathrm{FC}^i(0, T_N) = \delta \sum_{j=1}^{N+1} B^i(0, T_j) \mathbb{E}_{\mathbb{P}^{i, T_j}} \left[\left(L^i(T_{j-1}, T_{j-1}) - K^i \right)^+ \right]$$

Alternatively if we define $\widetilde{K}^i = 1 + \delta K^i$ (forward process approach)

$$\begin{aligned} \mathrm{FC}^i(0, T_N) &= \sum_{j=1}^{N+1} B^i(0, T_j) \mathbb{E}_{\mathbb{P}^{i, T_j}} \left[\left(1 + \delta L^i(T_{j-1}, T_{j-1}) - \widetilde{K}^i \right)^+ \right], \\ &= \sum_{j=1}^{N+1} C^i(0, T_j, \widetilde{K}^i) \end{aligned}$$

Pricing cross-currency derivates (2)

Numerical evaluation of the cap price

Define $X^i_{T_{j-1}}(t) = \int_0^t \lambda^i(s,T_{j-1})\,\mathrm{d}L_s^{i,T_j} = \ln \dfrac{1+\delta L^i(t,T_{j-1})}{1+\delta L^i(0,T_{j-1})}$

and let $\chi^{i,T_{j-1}}(z)$ be its characteristic function, then via a convolution representation

$$C^i(0,T_j,\tilde{K}^i) \;=\; B^i(0,T_j)\widetilde{K}^i\,\frac{\exp(\tilde{\xi}^i_j R)}{2\pi}\int_{-\infty}^{\infty}\exp(\mathrm{i}u\tilde{\xi}^i_j)\,\frac{\chi^{i,T_{j-1}}(\mathrm{i}R-u)}{(R+\mathrm{i}u)(1+R+\mathrm{i}u)}\,\mathrm{d}u$$

where $\tilde{\xi}^i_j = \ln(\widetilde{K}^i) - \ln(1+\delta L^i(0,T_{j-1}))$ and R is s.t. $\chi^{i,T_{j-1}}(\mathrm{i}R) < \infty$.

Pricing cross-currency derivates (3)

Cross-currency swaps

Floating-for-floating cross-currency $(i; \ell; 0)$ swap

Libor rate $L^i(T_{j-1}, T_{j-1})$ of currency i is received at each date T_j

Libor rate $L^\ell(T_{j-1}, T_{j-1})$ of currency ℓ is paid

Payments are made in units of the domestic currency

Thus the cashflow at time point T_j is (notional = 1)

$$\delta(L^i(T_{j-1}, T_{j-1}) - L^\ell(T_{j-1}, T_{j-1}))$$

Pricing cross-currency derivates (4)

The time-0-value of a floating-for-floating $(i;\ell;0)$ cross-currency forward swap in units of the domestic currency is

$$CCFS_{[i;\ell;0]}(0) = B^0(0,T_j)\left(\sum_{j=1}^{N+1}\frac{B^i(0,T_{j-1})}{B^i(0,T_j)}\exp\left(\mathcal{D}^i(0,T_{j-1},T_j)\right)\right.$$
$$\left.-\sum_{j=1}^{N+1}\frac{B^\ell(0,T_{j-1})}{B^\ell(0,T_j)}\exp\left(\mathcal{D}^\ell(0,T_{j-1},T_j)\right)\right)$$

where

$$\begin{aligned}\mathcal{D}^i&(0,T_{j-1},T_j)\\ &= -\int_0^{T_{j-1}}\lambda^i(s,T_{j-1})^\top c_s\zeta^i(s,T_j,T_{j+1})\,\mathrm{d}s\\ &\quad-\int_0^{T_{j-1}}\int_{\mathbb{R}^d}\left(\exp\left(\lambda^i(s,T_{j-1})^\top x\right)-1\right)\left(\overline{\zeta}_i(s,x,T_j,T_{j+1})-1\right)\nu_{0,T_j}(\mathrm{d}s,\mathrm{d}x)\end{aligned}$$

Pricing cross-currency derivates (5)

A quanto caplet with strike K^i, which expires at time T_{j-1}, pays at time T_j

$$QCpl^i(T_j, T_j, K^i) = \delta \overline{X}^i (L^i(T_{j-1}, T_{j-1}) - K^i)^+$$

where $\overline{X}^i$ is the preassigned foreign exchange rate

Time-0-value

$$\begin{aligned} QCpl^i(0, T_j, K^i) &= B^0(0, T_j) \mathbb{E}_{\mathbb{P}^{0,T_j}} [\delta \overline{X}^i (L^i(T_{j-1}, T_{j-1}) - K^i)^+] \\ &= B^0(0, T_j) \overline{X}^i \mathbb{E}_{\mathbb{P}^{0,T_j}} [(1 + \delta L^i(T_{j-1}, T_{j-1}) - (1 + \delta K^i))^+] \end{aligned}$$

(forward process approach)

Pricing cross-currency derivates (6)

Numerical evaluation of quanto caplets. Write

$$
\begin{aligned}
1+\delta L^i(T_{j-1},T_{j-1}) &= (1+\delta L^i(0,T_{j-1}))\exp\left(\int_0^{T_{j-1}} \lambda^i(s,T_{j-1})\, dL_s^{i,T_j}\right) \\
&= (1+\delta L^i(0,T_{j-1}))\exp\Big(\underbrace{\mathcal{M}^i(0,T_{j-1},T_j)}_{\text{assume density } \varrho}+\underbrace{\mathcal{D}^i(0,T_{j-1},T_j)}_{\text{non-random}}\Big)
\end{aligned}
$$

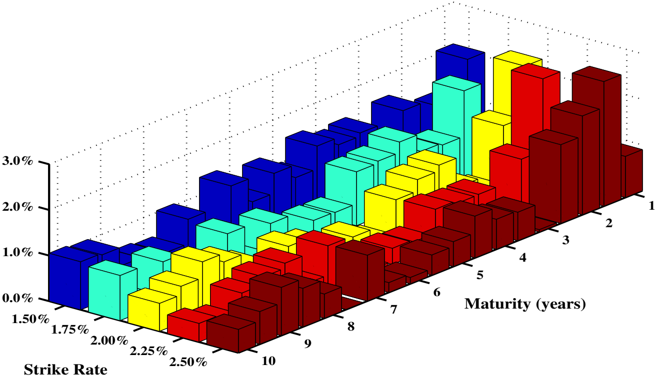
then for $v(x)=(e^{-x}-1)^+$

$$
QCpl^i(0,T_j,K^i)=B^0(0,T_j)\overline{X}^i(1+\delta K^i)(v*\varrho)(\xi_j)
$$

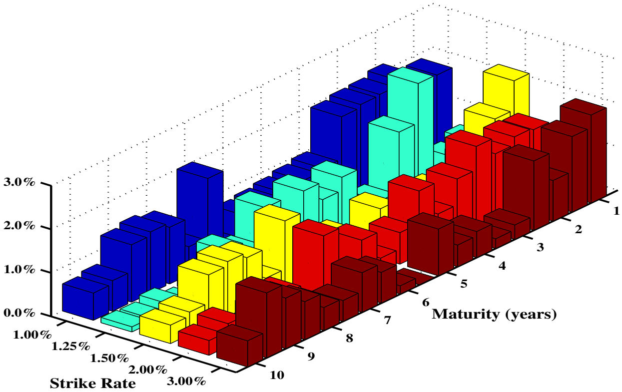
Finally we get

$$
\begin{aligned}
QCpl^i(0,T_j,K^i) &= B^0(0,T_j)\overline{X}^i(1+\delta K^i) \\
&\quad\cdot \frac{\exp(\xi_j R)}{2\pi}\int_{-\infty}^{\infty}\exp(\mathrm{i}u\xi_j)\frac{\chi^{\mathcal{M}^i,T_{j-1}}(\mathrm{i}R-u)}{(R+\mathrm{i}u)(R+1+\mathrm{i}u)}\, du
\end{aligned}
$$

Absolute errors of EUR caplet calibration



Absolute errors of USD caplet calibration



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