WEAKLY REARRANGEMENT INVARIANT SPACES AND APPROXIMATION BY LARGEST ELEMENTS

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1. Abstract

The abstract context of many problems in Approximation Theory can be described as follows. We are given a function space X, an increasing family of subsets $X_N \subset X, N = 1, ...,$ and the corresponding best approximation functional

$$\sigma_N(x)_X = \inf_{x^0 \in X_N} \left\| x - x^0 \right\|_X . \tag{1}$$

Given $N > 0$, we are interested in finding the near optimal sequence $x^{\mathrm{opt},N} \in X_N$ such that

$$\sigma_N(x)_X \approx \|x - x^{\mathrm{opt},N}\|_X, \tag{2}$$

where $\approx$ means equivalence up to constants indepdent of x and N.

This is interesting for many applications of approximation theory (e.g., to Numerical Analysis, Image Compression, PDE's, etc.), and, in such applications, another important aspect of finding a near optimal approximation x^{opt} in (2) is to provide an algorithm that is computationally simple and efficient.

Here we shall characterize the sequence spaces for which near optimal approximation can be achieved by selecting largest elements.

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