Author: Michael Frazier

Title: Global Estimates for Kernels of Neumann Series and Green's Functions

Abstract: This work is joint with Fedor Nazarov and Igor Verbitsky. Consider a linear operator T with operator norm $\|T\| < 1$. It is a standard fact that the Neumann series $I + T + T^2 + \ldots$ converges to the inverse of $I - T$. Suppose T is an integral operator on $L^2(\Omega)$, where Ω is a σ-finite measure space. Suppose the kernel K of T is non-negative, measurable, symmetric, and satisfies a certain quasi-metric condition (example: the Riesz potential). We show that the kernel H of $S = T + T^2 + T^3 + \ldots$ is bounded below by $Ke^{c_1 K_2/K}$ and above by $Ke^{c_2 K_2/K}$, where K_2 is the kernel of T^2, for constants $c_1, c_2 > 0$. As a consequence, we see, for example, that H is bounded by a constant multiple of K if and only if K_2 is bounded by a constant multiple of K.

We apply this result to obtain estimates for Green's functions associated with (possibly fractional) Schrödinger operators $(-\triangle)^{\alpha/2} - q$ for $q \geq 0$ and $0 < \alpha \leq 2$, on a domain Ω which could be all of $\mathbb{R}^n$, or a bounded domain in $\mathbb{R}^n$ which satisfies the boundary Harnack principle. These results can be restated as estimates for the conditional gauge of an α-stable process.

Solvability criteria for a certain non-linear equation of Ricatti type can be obtained as an application.