Challenges for the evaluation of the diagnostic imaging systems with nonlinear behavior

Lucretiu M. Popescu

Division of Imaging and Applied Mathematics
Center for Devices and Radiological Health
Food and Drug Administration
E-mail: lucretiu.popescu@fda.hhs.gov

February Fourier Talks
University of Maryland, College Park
February 21–22, 2013

Motivation

- Integral-geometry models used for image reconstruction are replaced by physical and statistical models
  - PET and SPECT already use iterative reconstruction algorithms with corrections for physical effects
  - X-ray Computed Tomography (CT) has started the transition to iterative reconstruction algorithms

Motivation

- Integral-geometry models used for image reconstruction are replaced by physical and statistical models
  - PET and SPECT already use iterative reconstruction algorithms with corrections for physical effects
  - X-ray Computed Tomography (CT) has started the transition to iterative reconstruction algorithms
- In CT there is a need to reduce the dose while maintaining diagnostic effectiveness

CT dose reduction estimation problem

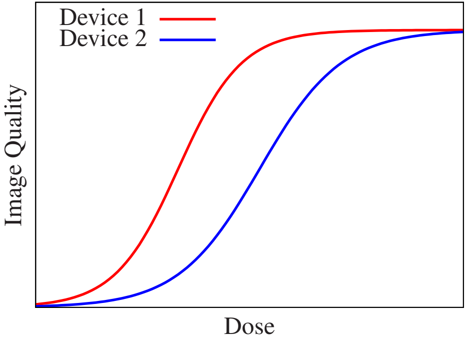
- The iterative reconstruction algorithms (IRA) promise improved image quality (IQ)

CT dose reduction estimation problem

- The iterative reconstruction algorithms (IRA) promise improved image quality (IQ)
- Need to determine an IQ metric related with diagnostic performance

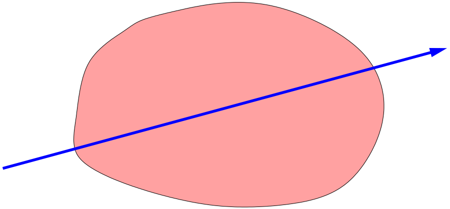
CT dose reduction estimation problem

- The iterative reconstruction algorithms (IRA) promise improved image quality (IQ)
- Need to determine an IQ metric related with diagnostic performance
- It should be a scalar, generate IQ vs. dose plots and find the equivalence points



Traditional CT image reconstruction

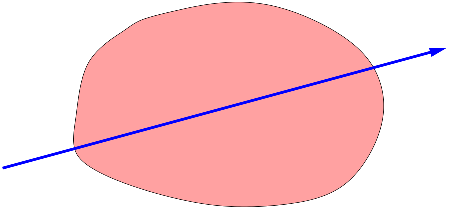
- Integral-geometry model



$$g(\mathbf{y}) = \int_{\mathcal{L}(\mathbf{y})} f(l)\mathrm{d}l$$

Traditional CT image reconstruction

- Integral-geometry model



$$g(\mathbf{y}) = \int_{\mathcal{L}(\mathbf{y})} f(l)\mathrm{d}l$$

- X-ray transmission tomography model

$$g_j = g_{0j} \mathrm{e}^{-\int_{\mathcal{L}_j} \mu(l)\mathrm{d}l} \Rightarrow \int_{\mathcal{L}_j} \mu(l)\mathrm{d}l = \log\left(\frac{g_{0j}}{g_j}\right)$$

where

g_{0j} data recorded without the object
g_j data recorded with the object

Discrete representation

- Projection

$$\mathbf{g} = H\mathbf{f}$$

Discrete representation

- Projection

$$\mathbf{g} = H\mathbf{f}$$

- Reconstruction

$$\mathbf{f} = H^{-1}\mathbf{g}$$

Discrete representation

- Projection

$$\mathbf{g} = H\mathbf{f}$$

- Reconstruction

$$\mathbf{f} = H^{-1}\mathbf{g}$$

- In the presence of noise

$$\hat{\mathbf{f}} = \mathbf{f} + \hat{\mathbf{n}}_f = H^{-1}(\mathbf{g} + \hat{\mathbf{n}}_g)$$

Discrete representation

- Projection

$$\mathbf{g} = H\mathbf{f}$$

- Reconstruction

$$\mathbf{f} = H^{-1}\mathbf{g}$$

- In the presence of noise

$$\hat{\mathbf{f}} = \mathbf{f} + \hat{\mathbf{n}}_f = H^{-1}(\mathbf{g} + \hat{\mathbf{n}}_g)$$

- The image quality is linearly determined by H^{-1} and $\hat{\mathbf{n}}_g$

Discrete representation

- Projection

$$\mathbf{g} = H\mathbf{f}$$

- Reconstruction

$$\mathbf{f} = H^{-1}\mathbf{g}$$

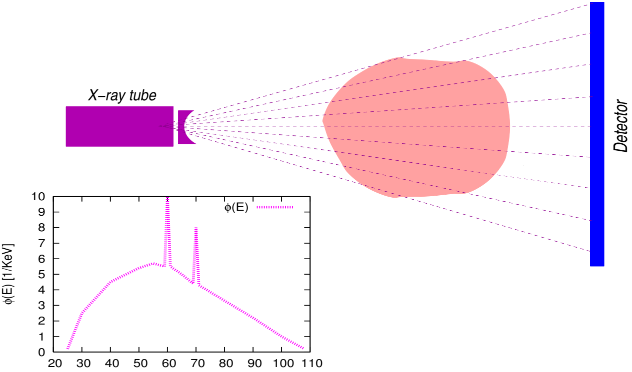
- In the presence of noise

$$\hat{\mathbf{f}} = \mathbf{f} + \hat{\mathbf{n}}_f = H^{-1}(\mathbf{g} + \hat{\mathbf{n}}_g)$$

- The image quality is linearly determined by H^{-1} and $\hat{\mathbf{n}}_g$
- Noise propagation is independent of the object (system property)

$$\hat{\mathbf{n}}_f = H^{-1}\hat{\mathbf{n}}_g$$

X-ray transmission tomography in real world



- Polychromatic source
- Attenuation dependent on energy. Scatter
- Energy integrating detectors, nonlinear response
- Statistical behavior

X-ray transmission tomography physical model

$$g_j = I \int \phi_j(E) \mathrm{e}^{-\int_{\mathcal{L}_j} \mu(l,E)\mathrm{d}l} \varepsilon_j(E) \xi(E) \mathrm{d}E + I s_j$$

where

- g_j the detector signal for projection j
- I the X-ray source intensity
- $\phi_j(E)$ the source spectrum
- $\mathrm{e}^{-\int_{\mathcal{L}_j} \mu(l,E)\mathrm{d}l}$ attenuation along the projection j
- $\varepsilon_j(E)$ detector efficiency
- $\xi(E)$ detector response signal; e.g. $\xi(E) \propto E$
- $I s_j$ scattered photons contribution

Iterative reconstruction algorithm

- The voxel's attenuation represented as $\mu_i(E) = f_i \mu_0(E)$
- Find the extreme value of a cost function

$$S(\underline{f}) = \sum_j \frac{(\hat{g}_j - g_j)^2}{\eta_j g_j} + \beta R(\underline{f})$$

$\beta R(\underline{f})$ regularization term, $R(\underline{f}) = \sum_i \sum_{k \in \mathcal{N}_i} \psi(f_i - f_k)$

Iterative reconstruction algorithm

- The voxel's attenuation represented as $\mu_i(E) = f_i \mu_0(E)$
- Find the extreme value of a cost function

$$S(\underline{f}) = \sum_j \frac{(\hat{g}_j - g_j)^2}{\eta_j g_j} + \beta R(\underline{f})$$

$\beta R(\underline{f})$ regularization term, $R(\underline{f}) = \sum_i \sum_{k \in \mathcal{N}_i} \psi(f_i - f_k)$

- Properties
  - Nonlinear behavior
  - Noise strongly dependent on the object
  - External constraints can be introduced

Image quality (IQ) measures

- Resolution
  - identify line or grid patterns
  - point spread function (PSF)
  - modulation transfer function, $\mathrm{MTF} = \mathcal{F}[\mathrm{PSF}]$

Image quality (IQ) measures

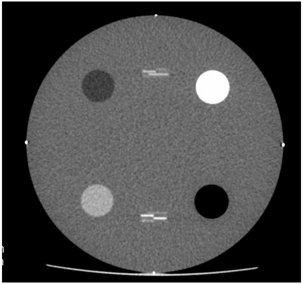
- Resolution
  - identify line or grid patterns
  - point spread function (PSF)
  - modulation transfer function, $\mathrm{MTF} = \mathcal{F}[\mathrm{PSF}]$
- Noise
  - pixel variance (no spatial correlations)
  - noise power spectrum (NPS)

Image quality (IQ) measures

- Resolution
  - identify line or grid patterns
  - point spread function (PSF)
  - modulation transfer function, $\text{MTF} = \mathcal{F}[\text{PSF}]$
- Noise
  - pixel variance (no spatial correlations)
  - noise power spectrum (NPS)

- For ranking we need to express the IQ as a single number

Contrast to noise ratio (CNR)



$$\text{CNR} = \frac{\text{ROI contrast}}{\text{pixel variance}}$$

- Does not account for spatial correlations of the noise
- Depends on the ROI original contrast
- Arbitrary scaling

Task based evaluation

- A test task relevant for the clinical application

Task based evaluation

- A test task relevant for the clinical application
- Yet simple enough
  - Can be analytically studied
  - Convenient to be applied experimentally

Task based evaluation

- A test task relevant for the clinical application
- Yet simple enough
  - Can be analytically studied
  - Convenient to be applied experimentally

Detection of small, low contrast, signals

Detection of a signal at known location

- We have
  - $\mathbf{g}_1$ – signal average
  - $\mathbf{g}_0$ – background average
  - K – noise covariance (same for signal and background)

Detection of a signal at known location

- We have
  - $\mathbf{g}_1$ – signal average
  - $\mathbf{g}_0$ – background average
  - K – noise covariance (same for signal and background)
- Likelihood ratio test for a given location $\hat{\mathbf{g}}$

$$\lambda(\hat{\mathbf{g}}) = \log\left[\frac{\text{Pr}(\hat{\mathbf{g}}|1)}{\text{Pr}(\hat{\mathbf{g}}|0)}\right] = (\mathbf{g}_0 - \mathbf{g}_1)^t K^{-1}\hat{\mathbf{g}}$$

If $\lambda(\hat{\mathbf{g}}) > \lambda_{\text{th}}$ then $\hat{\mathbf{g}}$ is declared positive

Detection of a signal at known location

- We have
  - $\mathbf{g}_1$ – signal average
  - $\mathbf{g}_0$ – background average
  - K – noise covariance (same for signal and background)
- Likelihood ratio test for a given location $\hat{\mathbf{g}}$

$$\lambda(\hat{\mathbf{g}}) = \log\left[\frac{\mathrm{Pr}(\hat{\mathbf{g}}|1)}{\mathrm{Pr}(\hat{\mathbf{g}}|0)}\right] = (\mathbf{g}_0 - \mathbf{g}_1)^t K^{-1}\hat{\mathbf{g}}$$

  If $\lambda(\hat{\mathbf{g}}) > \lambda_{\mathrm{th}}$ then $\hat{\mathbf{g}}$ is declared positive
- Signal to noise ratio (SNR)

$$d^2 = \frac{\{E[\lambda(\mathbf{g}_1)] - E[\lambda(\mathbf{g}_0)]\}^2}{\frac{1}{2}\{\mathrm{var}[\lambda(\mathbf{g}_1)] - \mathrm{var}[\lambda(\mathbf{g}_0)]\}}$$

Interpretation of SNR

- At high dose "noise" $\to 0 \Rightarrow$ SNR $\to \infty$

Interpretation of SNR

- At high dose "noise" $\rightarrow 0 \Rightarrow$ SNR $\rightarrow \infty$
- If we compare two modalities, then at high dose $\Delta\text{SNR} = \text{SNR}_2 - \text{SNR}_1$ can have arbitrary values

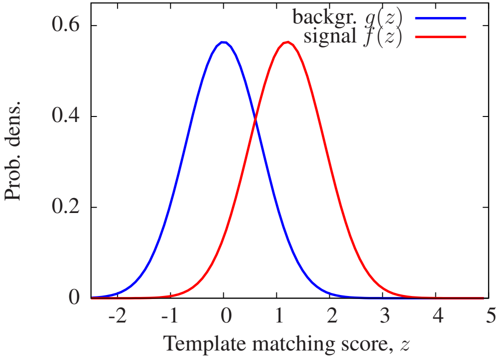
Interpretation of SNR

- At high dose "noise" $\rightarrow 0 \Rightarrow$ SNR $\rightarrow \infty$
- If we compare two modalities, then at high dose $\Delta\text{SNR} = \text{SNR}_2 - \text{SNR}_1$ can have arbitrary values
- SNR is not suited for direct quantitative comparisons

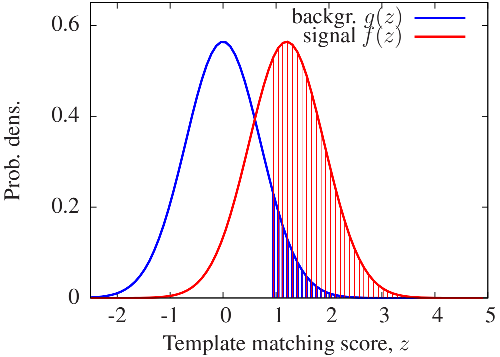
Interpretation of SNR

- At high dose "noise" $\rightarrow 0 \Rightarrow$ SNR $\rightarrow \infty$
- If we compare two modalities, then at high dose $\Delta\text{SNR} = \text{SNR}_2 - \text{SNR}_1$ can have arbitrary values
- SNR is not suited for direct quantitative comparisons
- We need to turn SNR into quantity that has a more direct connection with the signal detection performance

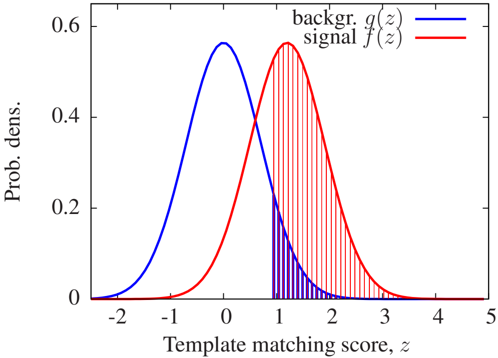
Relative operating characteristic (ROC)

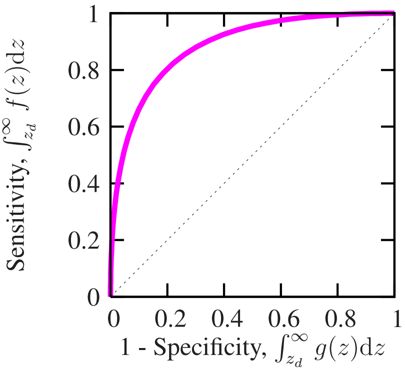


Relative operating characteristic (ROC)

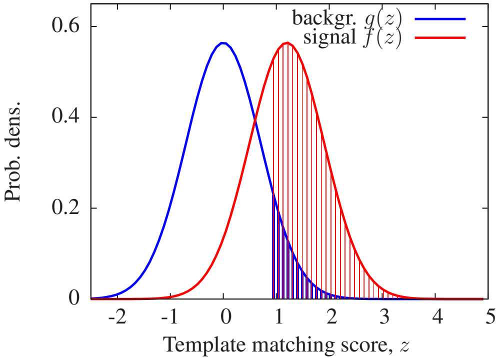


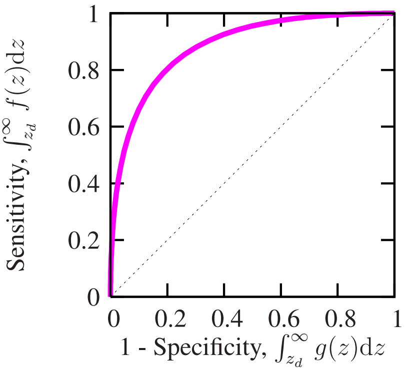
Relative operating characteristic (ROC)





Relative operating characteristic (ROC)



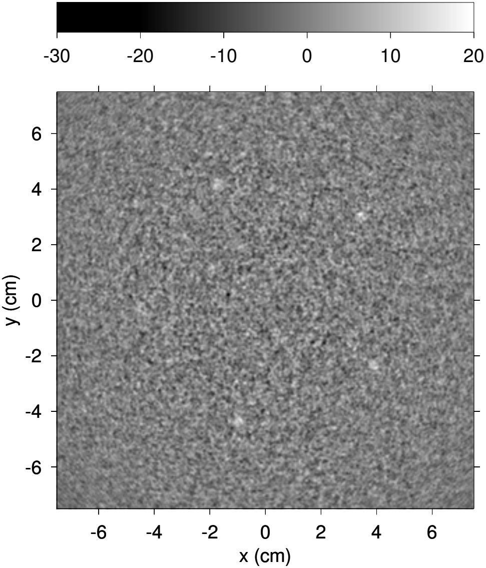


- Area under the ROC curve

$$A = \mathrm{Prob}\left(\text{signal score} > \text{background score}\right) \in (0.5, 1)$$

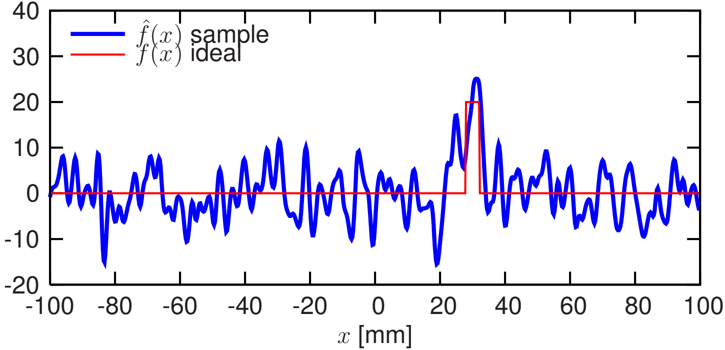
- Relation with SNR: $A = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{d}{2}\right)\right]$

Detection of signals at unknown locations

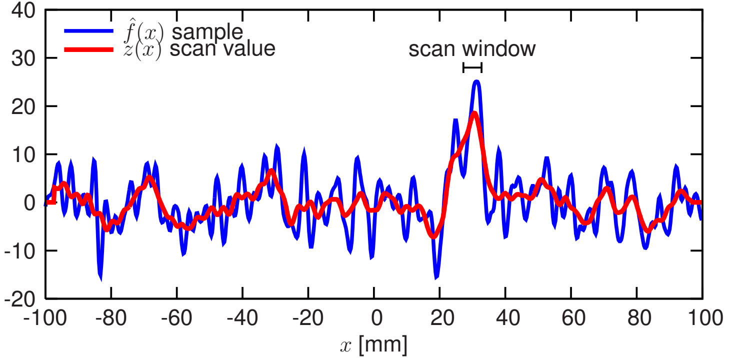


Detection of signals at unknown locations

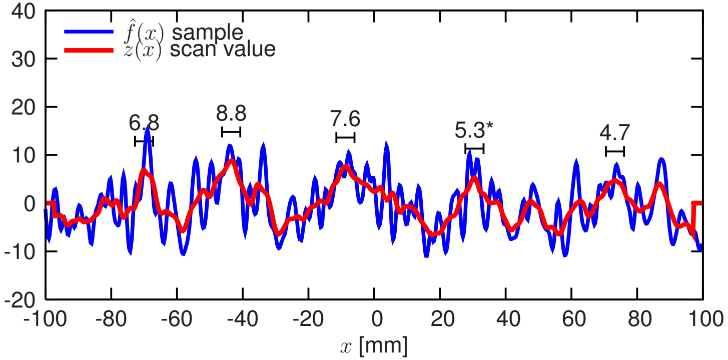
- One dimensional random field example



'Image' scanning

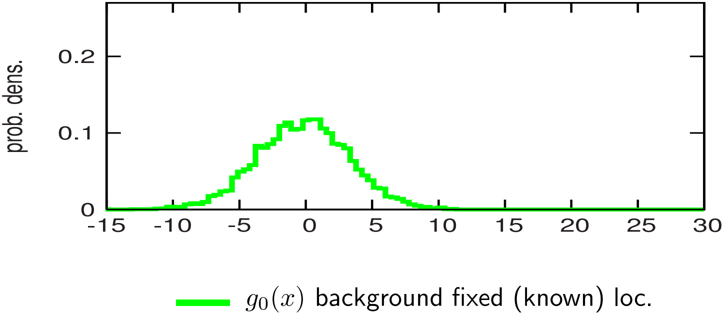


- Sometimes the signal scan-value is less than the background maximum

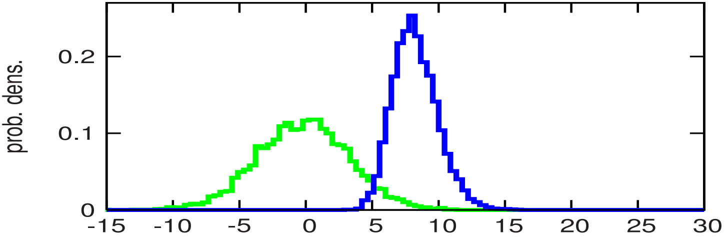


- Fraction of signals correctly localized $Q = 95\%$

Scan distributions



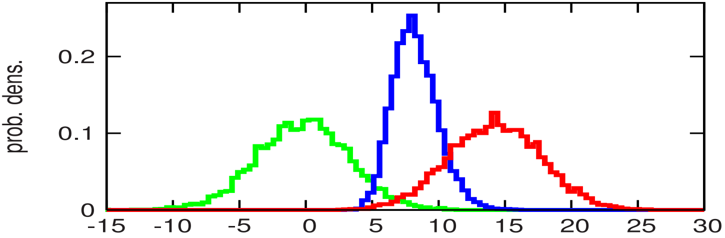
Scan distributions



$g_0(x)$ background fixed (known) loc.

$g(x)$ background maximum

Scan distributions

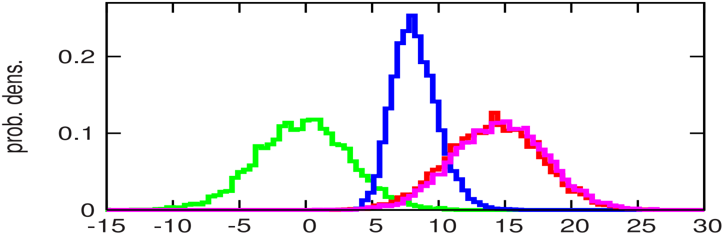


$g_0(x)$ background fixed (known) loc.

$g(x)$ background maximum

$s(x)$ signal fixed (known) loc.

Scan distributions



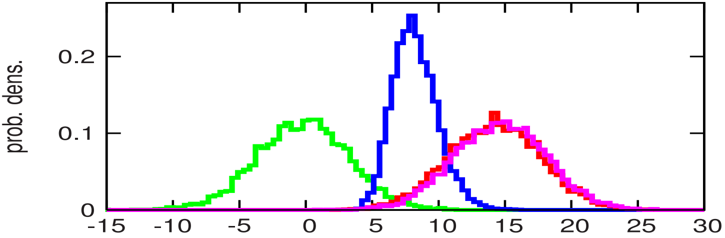
$g_0(x)$ background fixed (known) loc.

$g(x)$ background maximum

$s(x)$ signal fixed (known) loc.

$f(x)$ signal searched

Scan distributions



$g_0(x)$ background fixed (known) loc.

$g(x)$ background maximum

$s(x)$ signal fixed (known) loc.

$f(x)$ signal searched

- loc. known: SNR $= \frac{\langle z_s \rangle - \langle z_{g_0} \rangle}{\sqrt{\frac{1}{2}\left(\sigma_s^2 + \sigma_{g_0}^2\right)}} = 4.15$; ROC $A_0 = 0.998$
- loc. unknown: $Q = 0.95$, $Q_{\times 2} = 0.92$, $Q_{\times 4} = 0.88$, ...

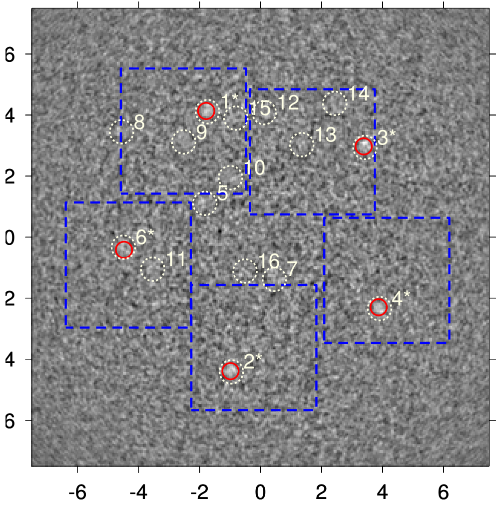
Signal known location vs. unknown location

- Signal known location
  - Does not account for the extreme background values
  - Requires signals with very low contrast in order to achieve moderately difficult detectability levels
  + Well modeled theoretically

Signal known location vs. unknown location

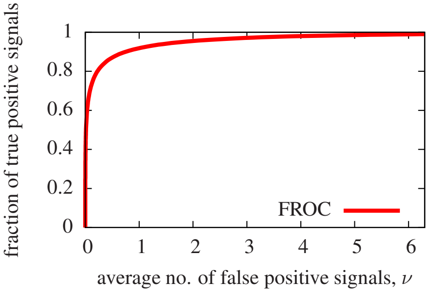
- Signal known location
  - – Does not account for the extreme background values
  - – Requires signals with very low contrast in order to achieve moderately difficult detectability levels
  - + Well modeled theoretically
- Signal unknown location
  - + More realistic for many clinical applications
  - + Allows for more reasonable signal contrast levels
  - – Difficult to model analytically, approximate solutions
  - + Practical approaches for signal searching and data analysis are available

Signal searching example

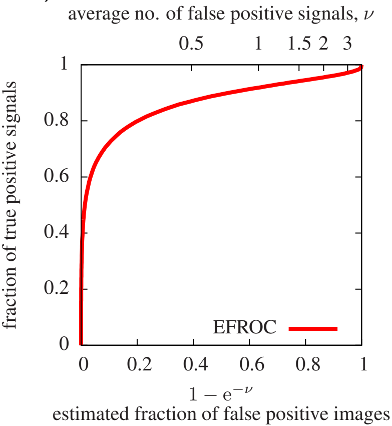


n	x	y	score	status
1	-1.74	4.11	7.48	true
2	-0.96	-4.41	6.67	true
3	3.42	2.94	5.91	true
4	3.90	-2.34	5.61	true
5	-1.83	1.11	4.56	
6	-4.50	-0.33	4.37	true
7	0.45	-1.38	4.36	
8	-4.56	3.45	3.91	
9	-2.52	3.12	3.67	
10	-0.99	1.95	3.56	
11	-3.54	-1.05	3.56	
12	0.12	4.08	3.56	
13	1.35	3.03	3.37	
14	2.43	4.38	3.27	
15	-0.81	3.90	3.12	
16	-0.51	-1.11	3.09	

Exponential transformation of the FROC (EFROC) (Popescu, *Med. Phys.*, 2011)

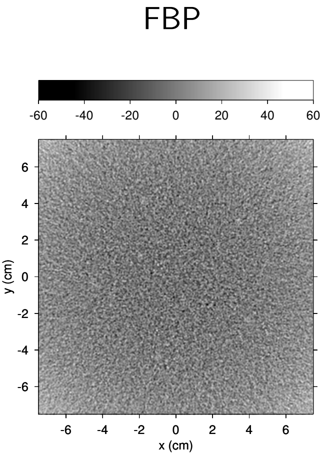


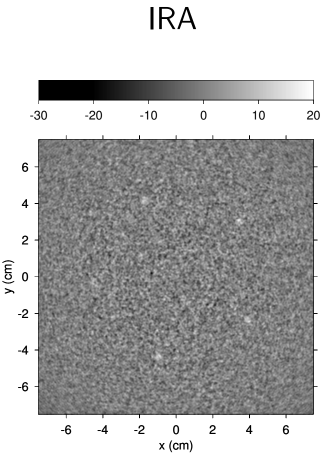
⇒



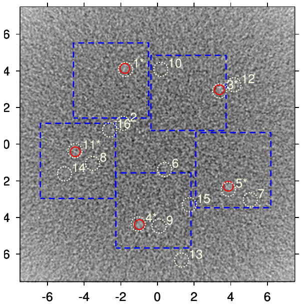
- AUC estimation: $\hat{A}_{\mathsf{FE}} = \frac{1}{I}\sum_{i=1}^{I} \mathrm{e}^{-\frac{1}{N}\sum_{j=1}^{J} H(y_j - x_i)}$ $\quad H(z) = \begin{cases} 1\,; & z > 0 \\ \frac{1}{2}\,; & z = 0 \\ 0\,; & z < 0 \end{cases}$
- $N = \frac{\text{total area scanned}}{\text{reference area}}$

Filtered Back Projection vs. Iterative Reconstruction



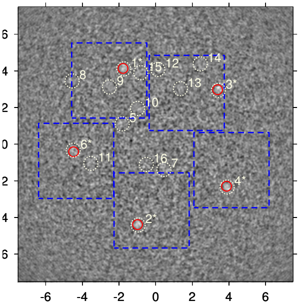


FBP



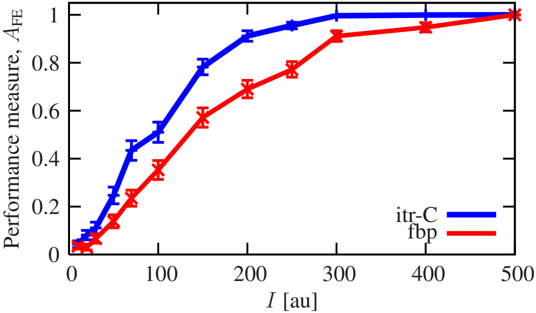
n	x	y	score	status
1	-1.74	4.08	9.25	true
2	-1.89	1.14	7.91	
3	3.39	2.91	7.83	true
4	-1.05	-4.35	6.89	true
5	3.87	-2.46	6.72	true
6	0.39	-1.38	6.35	
7	5.07	-3.03	5.98	
8	-3.54	-1.05	5.82	
9	0.09	-4.47	5.80	
10	0.18	4.08	5.76	
11	-4.50	-0.36	5.76	true
12	4.17	3.24	5.63	
13	1.32	-6.33	5.56	
14	-5.07	-1.62	5.51	
15	1.74	-3.30	5.48	
16	-2.61	0.78	5.29	

IRA



n	x	y	score	status
1	-1.74	4.11	7.48	true
2	-0.96	-4.41	6.67	true
3	3.42	2.94	5.91	true
4	3.90	-2.34	5.61	true
5	-1.83	1.11	4.56	
6	-4.50	-0.33	4.37	true
7	0.45	-1.38	4.36	
8	-4.56	3.45	3.91	
9	-2.52	3.12	3.67	
10	-0.99	1.95	3.56	
11	-3.54	-1.05	3.56	
12	0.12	4.08	3.56	
13	1.35	3.03	3.37	
14	2.43	4.38	3.27	
15	-0.81	3.90	3.12	
16	-0.51	-1.11	3.09	

FBP and IRA performance as function of dose



- The results obtained from 20 signal-present and 20 signal-absent image samples

Conclusions

- The nonlinear behavior limits the use of traditional image quality metrics

Conclusions

- The nonlinear behavior limits the use of traditional image quality metrics
- We have to use task-based evaluations

Conclusions

- The nonlinear behavior limits the use of traditional image quality metrics
- We have to use task-based evaluations
- Detection of small signals at unknown locations proves to be a versatile approach

Conclusions

- The nonlinear behavior limits the use of traditional image quality metrics
- We have to use task-based evaluations
- Detection of small signals at unknown locations proves to be a versatile approach
- Confirmed that IRA is better than FBP for the studied case

Conclusions

- The nonlinear behavior limits the use of traditional image quality metrics
- We have to use task-based evaluations
- Detection of small signals at unknown locations proves to be a versatile approach
- Confirmed that IRA is better than FBP for the studied case
- Future work
  - Refine signal searching algorithms
  - Signals of different sizes and shapes
  - Compare with human observers performance

Thank you

Acknowledgments

- Brandon Gallas
- Kyle Myers
- Nicholas Petrick
- Berkman Sahiner
- Frank Samuelson
- Rongping Zeng